Jonathan Bennett
University of Birmingham
Tight induction on scales and the nonlinear Brascamp–Lieb inequality

We describe an efficient use of the method of induction-on-scales in the context of nonlinear perturbations of the Brascamp–Lieb inequality. Central to our approach is the specific use of gaussians to pass between scales, resulting in certain near-monotonicity statements under certain approximate heat flows. As an application we establish a near-optimal local version of Young's convolution inequality on a general Lie group, answering a question of Cowling, Martini, Müller and Parcet. This talk is based on work with Neal Bez, Stefan Buschenhenke, Michael Cowling and Taryn Flock, with further contributions by Rory Duncan.

Anthony Carbery
University of Edinburgh
A multilinear Maurey-type 'factorisation' theorem

We discuss a multilinear analogue of a classical factorisation theorem of B. Maurey which states that a positive linear operator mapping a Banach lattice into L^q for $0 < q < 1$ factorises through L^1. Our point of view sheds light on Guth's technique for obtaining the endpoint multilinear Kakeya theorem. This is joint work with Stefán Valdimarsson but the perspective has been very much inspired by conversations with Timo Hänninen.

Michael Christ
University of California, Berkeley
Cases of equality in certain multilinear inequalities

The Brascamp–Lieb–Luttinger inequalities (first stated by Rogers) state that certain multilinear functionals defined by integrals over Euclidean spaces attain their maximum values, over tuples of functions with specified distribution functions, when the functions involved are radial and nondecreasing. We show for indicator functions of sets that under certain hypotheses on their measures, these are the only cases of equality, up to natural symmetries. The principal hypotheses are (strict) admissibility and for dimension one, a certain genericity.

The result had been known in a few special cases, principally that of the Riesz–Sobolev inequality (Burchard).

The strategy entails establishing a more quantitative form of this uniqueness with an optimal exponent. The one-dimensional case has a special structure and role, and is treated first. The analysis for higher dimensions is joint work with Kevin O'Neill. It relies on Steiner symmetrization, the one-dimensional result, a perturbative expansion, and spherical harmonics.

Michael G. Cowling
University of New South Wales
bmo and product quasiconformal maps

In 1975, Reimann proved that changes of variable in $\mathbb{R}^n$ preserve BMO if and only if they are quasiconformal. More precisely, if ϕ is a homeomorphism of $\mathbb{R}^n$, then $\|f\|_{\mathrm{BMO}} \eqsim \|f\circ\phi\|_{\mathrm{BMO}}$ if and only if ϕ is quasiconformal. This has been extended to more general metric measure spaces (satisfying appropriate hypotheses) by various authors.

In this talk, we consider the analogous results for product spaces. In summary, using bmo and with an appropriate definition of product conformality, which in particular implies that they are product maps, possibly composed with a simple exchage, Reimann's theorem extends to products $\mathbb{R}^m \times \mathbb{R}^n$; in more general product spaces the difficulties to overcome are finding the right hypotheses and extending already technical results to a more general context.

Gustavo Garrigós
Universidad de Murcia
Bergman projections in tubes over cones

In this talk we will survey various problems related with the L^p boundedness of Bergman projections in tube domains over cones. In particular, for light-cones, we discuss the solution of the main conjecture based on the sharp decoupling inequalities of Bourgain and Demeter, and recent extensions like the complete range for weighted Bergman projections, which settles results started by Debertol. We will also examine problems that remain open, even in the setting of light-cones.

Tuomas Hytönen
University of Helsinki
Of commutators and Jacobians

The L^p boundedness of commutators $[b,T] = bT - Tb$ of pointwise multiplication b and singular integral operators T has been well studied for a long time. There are also many results about L^p to L^q boundedness for $p < q$, but (until recently) almost nothing for $p > q$. I will supply the missing pieces to present a complete picture of the L^p to L^q boundedness for all $p, q \in (1,\infty)$, and relate the regime of exponents $p > q$ to the mapping properties of the Jacobian on first order Sobolev spaces.

Izabella Łaba
University of British Columbia
Decoupling and differentiation for Cantor-type measures on the line

A result of Laba and Pramanik (2009) says that there exist measures on the line, supported on Cantor-type sets of dimension less than 1, that differentiate L^p spaces for a non-trivial range of p. We discuss an alternative approach based on decoupling that produces a weaker result but covers new classes of measures.

Loredana Lanzani
Syracuse University
— on behalf of Elias M. Stein, *Princeton University* —
On regularity and irregularity of the Cauchy–Szegő projection in several complex variables

It is known that for domains $D \Subset \mathbb{C}^n$ that are of class C^2 and are strongly pseudo-convex, the Cauchy–Szegő projection is bounded in $L^p(\mathrm{b}D, d\Sigma)$ for $1 < p < \infty$. (Here $d\Sigma$ is induced Lebesgue measure.) We show, using appropriate worm domains, that this fails for any $p \neq 2$, when we assume that the domain in question is only weakly pseudo-convex. Our starting point are the ideas of Kiselman–Barrett introduced more than 30 years ago in the analysis of the Bergman projection. However the study of the Cauchy–Szegő projection raises a numer of new issues and obstacles that need to be overcome. We will also compare these results to the analogous problem for the Cauchy–Leray integral, where however the relevant counter-example is of much simpler nature.

Mariusz Mirek
University of Wrocław
Jump inequalities in harmonic analysis

2-jump inequalities can be thought of as endpoint estimates for r-variational inequalities for $r = 2$. We will show that jump quasi-seminorms can be identified with suitable interpolation spaces. This is a very useful observation, which has many interesting consequences in classical and discrete harmonic analysis. This is joint project with E.M. Stein and P. Zorin-Kranich.

Detlef Müller
Christian-Albrechts-Universität zu Kiel
On Fourier restriction for hyperbolic surfaces

In contrast to what is known about Fourier restriction for elliptic surfaces, our understanding of the corresponding question for hyperbolic surfaces is still very rudimentary, with the exception of the "saddle" $z = xy$. In this talk, I shall report on recent progress on this problem achieved in joint work with S. Buschenhenke and A. Vargas.

Alexander Nagel
University of Wisconsin–Madison
Geometric estimates of Bergman kernels for certain tube domains

We show that in a number of cases the Bergman kernel for a tube domain over a convex set E can be estimated in terms of the volumes of appropriate caps cut off from the convex set by hyperplanes. In particular we study bounded convex sets, unbounded convex sets above the graph of a convex polynomial, and certain convex cones. The results are related to joint work with Malabika Pramanik.

Marco M. Peloso
Università degli Studi di Milano
Potential spaces on Lie groups

Let G be a connected Lie group G and X be a family of linearly independent left-invariant vector fields on G satisfying Hörmander's condition. Let χ be a positive character of G and consider the measure μ_χ whose density with respect the right Haar measure ρ is χ. In this talk we introduce Sobolev spaces $L^p_s(\mu_\chi)$ adapted to X and μ_χ, and study embedding theorems and algebra properties of these spaces. We also consider the case of Besov spaces. We apply these results to some nonlinear heat and Schrödinger equations. This talk is based on work in collaboration with Tommaso Bruno, Anita Tabacco and Maria Vallarino.

Malabika Pramanik
University of British Columbia
Restriction of Laplace–Beltrami eigenfunctions to fractals

There is a long line of research addressing various aspects of the following question: given a compact, smooth Riemannian manifold, how do the eigenfunctions of the Laplace–Beltrami operator "concentrate"? In 2007, Burq, Gerard and Tzvetkov obtained growth estimates for Lebesgue norms of these eigenfunctions when restricted to a smooth embedded submanifold. How do these estimates change when the eigenfunctions are restricted to still sparser sets of fractal type? We report on ongoing joint work in this direction with Suresh Eswarathasan.

Andreas Seeger
University of Wisconsin–Madison
Riesz means of Fourier series and integrals: Strong summability at the critical index

We consider Riesz means at the critical index, for Fourier series and integrals in higher dimensions. As almost everywhere convergence may fail we ask what can be said about the concept of (q-)strong convergence a.e. (or strong summability) which is weaker than a.e. convergence. Joint work with Jongchon Kim.

Christopher P. Sogge
Johns Hopkins University
Sharp local smoothing estimates for Fourier integral operators

We present joint work with D. Beltran and J. Hickman on sharp local smoothing estimates for general Fourier integral operators.

Terence Tao
UC Los Angeles

An inverse theorem for the Kneser inequality and applications

A theorem of Kneser asserts that if A, B are measurable subsets of a compact group with Haar measure μ, then the measure of the product set AB is at least $\min(\mu(A) + \mu(B), 1)$. There is also a more quantitative version involving the level sets of convolutions $1_A * 1_B$. The result can be proven using submodularity inequalities. In the case of an abelian group, we prove an inverse theorem that asserts, roughly speaking, that equality or near-equality can only hold if A and B are essentially "parallel Bohr sets" — pullbacks of intervals under continuous homomorphisms into the unit circle. In joint work with Joni Teravainen, we use this result to obtain some new number theoretic consequences about sign patterns of multiplicative functions.

Sundaram Thangavelu
Indian Institute of Science

Hardy's inequality for the sublaplacian on H-type groups

In this talk I plan to describe my recent results (obtained jointly with Luz Roncal) on the extension problem associated to sublaplacians on H-type groups and Hardy type inequalities for fractional powers of the sublaplacians. Solutions of the extension problem are written down explicitly and used to establish a trace Hardy inequality from which we deduce Hardy inequality with sharp constants.

Christoph Thiele
Universität Bonn

Recent results on singular Brascamp–Lieb type inequalities

Singular Brascamp–Lieb integrals arise when one of the functions in a typical Brascamp–Lieb datum is replaced by a Calderon–Zygmund singular integral kernel. Unlike the well known theory for Brascamp–Lieb integrals, we are far from understanding L^p boundedness of singular Brascamp–Lieb integrals. A few special cases are understood, and we report on some of the latest developments on the matter.

Xavier Tolsa
ICREA – Universitat Autònoma de Barcelona
Favard length, analytic capacity, and the Cauchy transform

In 1960's Vitushkin conjectured that a compact set in the plane is non-removable for bounded analytic functions (or equivalently, has positive analytic capacity) if and only it has positive Favard length, or in other words, its orthogonal projections have positive length in a set of directions of positive measure. In 1986 Mattila showed that this conjecture is false. However, it is not known yet if one of the implications in Vitushkin's conjecture holds. Namely, does positive Favard length imply positive analytic capacity? In this talk I will present a joint result with Alan Chang related to this open question. In a sense, this asserts that if one strengthens the assumption of positive Favard length in a suitable way, then the answer is positive. More precisely, if the density of a projected measure is in L^2 in an interval of directions, then the Cauchy transform with respect to this measure is bounded in L^2 and analytic capacity is positive.

Ana Vargas
Universidad Autónoma de Madrid
On a bilinear Bochner–Riesz operator

We consider a bilinear Bochner–Riesz operator defined by Bernicot–Grafakos–Song–Yan. We make use of a decomposition that relates this operator to the square function associated to the classical Bochner–Riesz multiplier. As a consequence, we obtain new results for the bilinear operator. This is a joint work with Eunhee Jeong and Sanghyuk Lee.

Joan Verdera
Universitat Autònoma de Barcelona
Minimizers of energies related to dislocation theory

Models for dislocation theory lead to the problem of minimizing the energy of a probability measure arising from the planar "interaction" kernel $-\log|z| + x^2/|z|^2, z = x + iy$, and a "confinement" $|z|^2/2$. The unique minimizer is known to be the measure on the vertical segment $[-i\sqrt{2}, i\sqrt{2}]$ with density $\frac{1}{\pi}\sqrt{2-y^2}$ (the semi-circle law). We study minimizers of the energy obtained by introducing a real parameter α in the interaction kernel: $-\log|z| + \alpha\, x^2/|z|^2$. For $\alpha = 0$ Frostman proved that the unique minimizer is the normalized characteristic function of the unit disc. We prove that for α between -1 and 1 the unique minimizer is the normalized characteristic function of an ellipse with semiaxes determined by α. This is joint work with J.A. Carrillo, J. Mateu, M.G. Mora, L. Rondi and L. Scardia.

Alexander Volberg
Michigan State University

From harmonic analysis on trees to harmonic analysis on hypercube

By now classical results of Bollobàs, Burkholder–Gundy, and Davis give an approach to obtaining sharp estimates for martingale square function via the initial martingale in various function spaces such as L^p or weak L^1, or spaces with exponential integrability. One can observe that all these results are equivalent to solving a certain (and always the same) PDE obstacle problem, where only the obstacle got changed from problem to problem. The next observation is that one can "dualize" this obstacle problem. The result is another obstacle problem that now gives a corresponding harmonic analysis result (or Poincaré type inequality) on hyper cube (and, thus, in gaussian space). As examples of this approach we can list the log-Sobolev inequality as dual to Chang–Wilson–Wolff estimate, or hypercube isoperimetric inequality as being dual to Bollobàs estimate of weak norm of square function of dyadic martingale.

James R. Wright
University of Edinburgh

Strongly singular integrals on stratified Lie groups

We revisit the class of oscillating spectral multipliers on Lie groups and address endpoint bounds. More importantly we begin to address the issue of what are the endpoint bounds. This is connected to the problem of seeking minimal smoothness conditions for Mikhlin–Hörmander spectral multipliers as well as the analysis of the wave equation on these groups. This is joint work with Paolo Ciatti.