Some open problems

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Definitions

Definition

1. Turing determinacy (TD) says that for every set A of *Turing degrees*, either A or the complement of A contains an upper cone.
2. Strong Turing determinacy (sTD) says that for every set A of *reals* ranging Turing degrees cofinally, A ranges an upper cone of Turing degrees.

TD is more natural than AD.

Some results

Theorem (Martin)

Over ZF, AD implies sTD + $CC_{\mathbb{R}}$ and so TD. $CC_{\mathbb{R}}$ means the axiom of countable choice for sets of reals.

Theorem (Woodin)

Over ZF + DC, sTD implies the "regular" properties for sets of reals.

Theorem (Peng and Y)

Over ZF, TD implies $CC_{\mathbb{R}}$.

Some open problems in set theory

Question

Over ZF,

1. *Does TD(+DC) imply AD? (Woodin proves that* $L(\mathbb{R}) \models TD + DC + ZF \to AD$.*)*
2. *Does* $TD + DC$ *imply the regular properties for sets of reals?*

Our question

Question

Over ZF,

1. *Does TD imply $DC_{\mathbb{R}}$?*
2. *Does TD(+DC) imply sTD?*

For question (1), what we have known is that $ZF + TD$ implies a measure theory version of $DC_{\mathbb{R}}$. I.e. if the relation R has the property that for any x, the set $\{y \mid R(x, y)\}$ has a positive inner measure.