On hypotheses under which P=NP

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Contents

Research problems (1/2)

1. There are well-known hypotheses under wich $P = NP$. For Xample, if $SAT \in P$ then $P = NP$, if SAT is P-selective then $P = NP$, etc. Is it possible to compare difficulties of these mathematical statements?

Research problem (2/2)

2. Is it possible to define a world (= an interpretation) in which each hypothesis is true?

Motivation of the research (1/2)

1. Consider the following statements.

(1) $SAT \in P$

(2) $SAT \in P\text{-sel}$, in which P-sel is the set of all P-selective sets.

It is known that $P \subseteq P\text{-sel}$, but P-sel is not a subset of P. Therefore, (2) is easier to prove than (1). But, is this intuition correct? Is there a way by which the difficulty of a statement can be measured?

Motivation of the research (2/2)

2. There are structures in which $P=NP$.

(1) P versus NP and computability theoretic constructions in complexity theory over algebraic structures, Gunther Mainhardt, The Journal of Symbolic Logic, Vol 69, No 1, March 2004

(2) A structure with $P=NP$, Christine Gaßner, Computer Science Report Series, University of Wales Swansea, CSR 7, 2006

These are mathematical "worlds". In these worlds both $SAT \in P$ and SAT is p-selective are true. But is it possible to define a "physical" world in which these statements are true?

Related work (1/3)

1. Algorithmic Complexity of Mathematical problems: An overview of Results and Open problems, Cristian S. Calude, Elena Calude, Int. Journal of Unconventional Computing, Vol 9, 2013

(1) The complexity of a Π_1 problem π $= \forall n\, P(n)$: the size of the smallest program that systematically searches for a counter-example for π

Related work (2/3)

(2) A formal definition of $C_U(\pi)$

U: a universal prefix-free Turing machine

π: Π_1 problem

Π_p: $\inf\{n : p(n) = \text{false}\}$

the program that tries to find the smallest n such that $p(n)$ is false

The complexity of a Π_1 problem π (with respect to U)

- $C_U(\pi) = \min\{\,|\Pi_p| : \pi = \forall n\, p(n)\}$

Related work (3/3)

(3) $Cu(\pi)$ is not computable.

Classification of Π_1 problems

$$C_{u,n} = \{ \pi \mid \pi \text{ is a } \Pi_1 \text{ problem}, \; Cu(\pi) \le 2^{10} \cdot n \}$$

Some known results

(The complexity of mathematical statements, Melissa S. Queen, https://www.cs.dartmouth.edu/~ac/Pubs/Students/mq-thesis-final.pdf)

Legendre's Conjecture

Fermat's Last Theorem

Goldbach's Conjecture

⋮

Current ongoing work (1/5)

Candidate hypotheses (not exhaustive)

SAT $\in$ P.

SAT is P-selective.

There is an NP complete sparse language.

There is an NP complete tally language.

There is an oracle for TQBF.

⋮

Current ongoing work (2/5)

SAT $\in$ P

$\forall x\, p(x)$, where

x: a boolean formula

$p(x)$: a predicate that is true if x is decidable in polynomial time, no otherwise

Construct Π_{SAT}

Current ongoing work (3/5)

$p(x)$ is semidecidable – for all possible pairs of programs p, and polynomials q, determine whether x is decided correctly by p in q steps. (Using the technique in Inductive complexity of the P versus NP problem, Christian S. Calude, Elena Calude, Melissa S. Queen, Parallel Processing Letters, 2013)

Current ongoing work (4/5)

Consider the following hypotheses again.

① $SAT \in P$. ② SAT is P-selective.

③ There is an NP complete sparse language.

④ There is an NP complete tally language.

⑤ There is an oracle for TQBF.

Is it possible to select one that looks almost consistent with physical properties that we experience every day?

Current ongoing work (5/5)

We can define a formal system in which each is an axiom. Then, we can have different worlds (= formal systems). Each world is characterized by the set of all theorems provable in each formal system.

Conclusion and future directions

1. It is possible to rank different hypotheses under which $P = NP$ according to their difficulties.

2. There may be diffent worlds (= formal systems) in which $P = NP$.

3. Is it possible to select a world that is almost consistent with physical properties that we experience everyday?