Tolerance-Based Techniques for Approximate Reasoning

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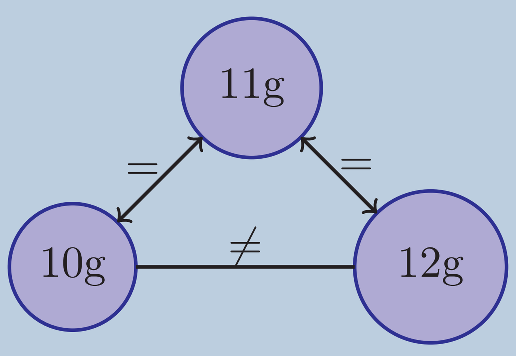
Introduction

- Reasoning with incomplete, imperfect information is very common in human communication.
- Its modeling is a highly nontrivial task, and remains an important issue in applications of artificial intelligence.
- Various notions are associated to such information: **uncertainty**, **imprecision**, **vagueness**, **fuzziness**.
- Different methodologies have been proposed to deal with them: **default logic**, **probability**, **fuzzy sets**, etc.
- For many problems in this area, exact equality is replaced by its approximation.
- Idea by **Poincaré**: in physical world, accumulation of measurement errors lead to the violation of transitivity of equality (in contrast to the ideal mathematical world).
- **Example**: experiment with small weights in measuring the **differential sensitivity** (*Fechner's* weight-lifting experiment)



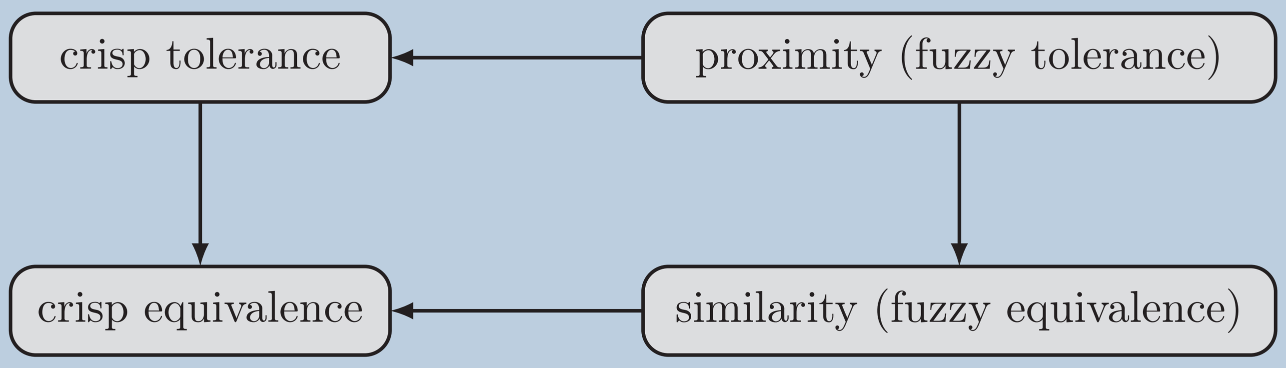
Tolerance Relation

- In the original version, tolerance relations were crisp: *two objects are either close to each other or not.*
- Later, their graded counterparts appeared which led, among others, to tolerance relations in the fuzzy setting.
- For a set S, a mapping $\mathcal{R}$ from $S \times S$ to the real interval $[0,1]$ is called a binary **fuzzy relation** on S.
- By fixing a number λ, $0 \le \lambda \le 1$, we can define a crisp counterpart of $\mathcal{R}$, named the **λ-cut** of $\mathcal{R}$ on S, as $\mathcal{R}_\lambda := \{(s_1, s_2) \mid \mathcal{R}(s_1, s_2) \ge \lambda\}$.
- A fuzzy relation $\mathcal{R}$ on a set S is called a **proximity relation** (also known as a **fuzzy tolerance relation**) on S iff it is reflexive and symmetric in the following sense:

  Reflexivity: $\mathcal{R}(s,s) = 1$ for all $s \in S$;

  Symmetry: $\mathcal{R}(s_1,s_2) = \mathcal{R}(s_2,s_1)$ for all $s_1, s_2 \in S$.
- A λ-cut of a proximity relation is a crisp tolerance relation.
- A T-norm $\wedge$ is an associative, commutative, non-decreasing binary operation on $[0,1]$ with 1 as the unit element.
- A proximity relation (on S) is called a **similarity relation** (also known as a **fuzzy equivalence relation**) (on S) iff it is transitive in the following sense:

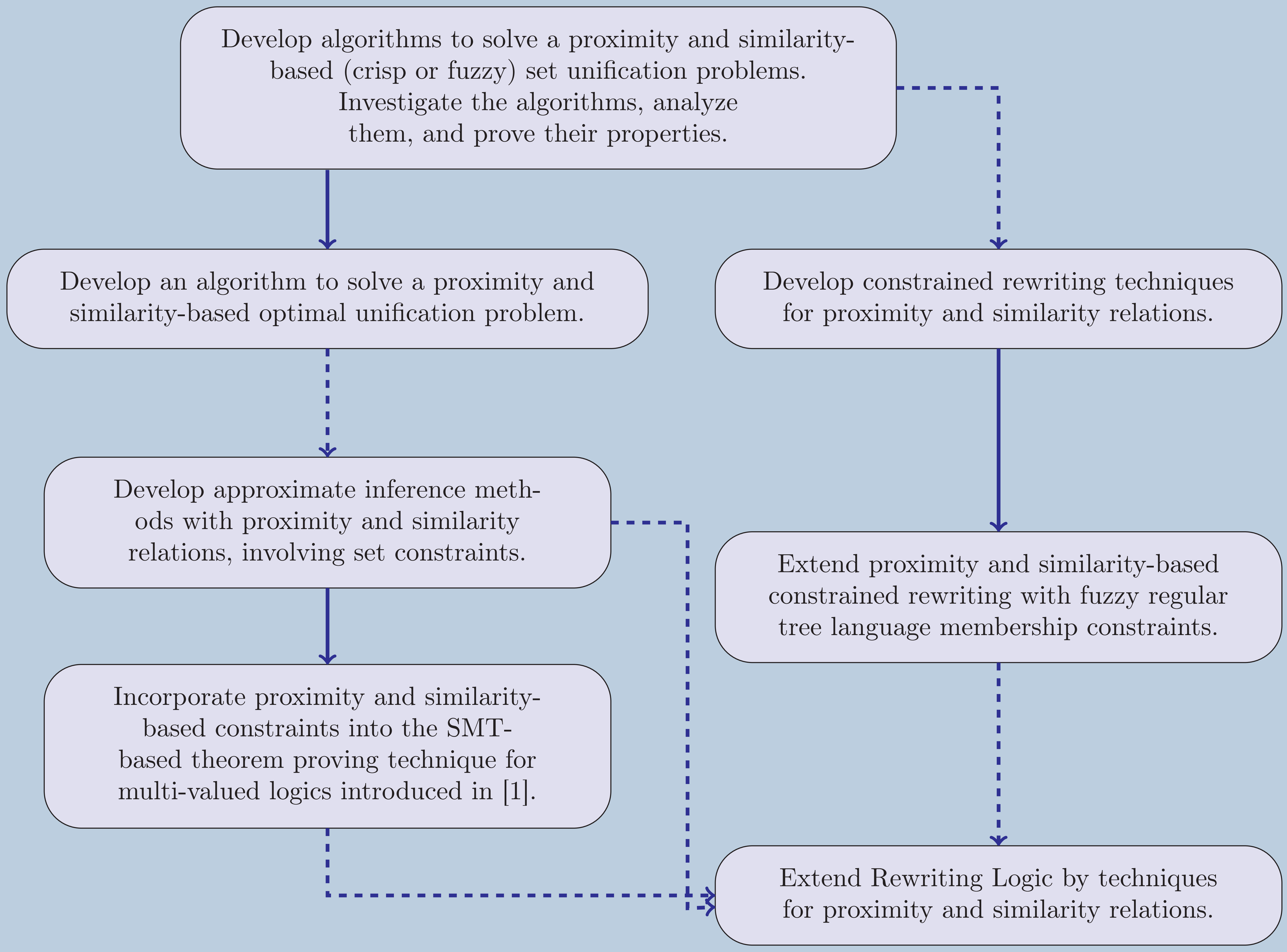
  Transitivity: $\mathcal{R}(s_1,s_2) \ge \mathcal{R}(s_1,s) \wedge \mathcal{R}(s,s_2)$ for any $s_1, s_2, s \in S$.
- A λ-cut of a similarity relation is a crisp equivalence relation.



Problem Statement

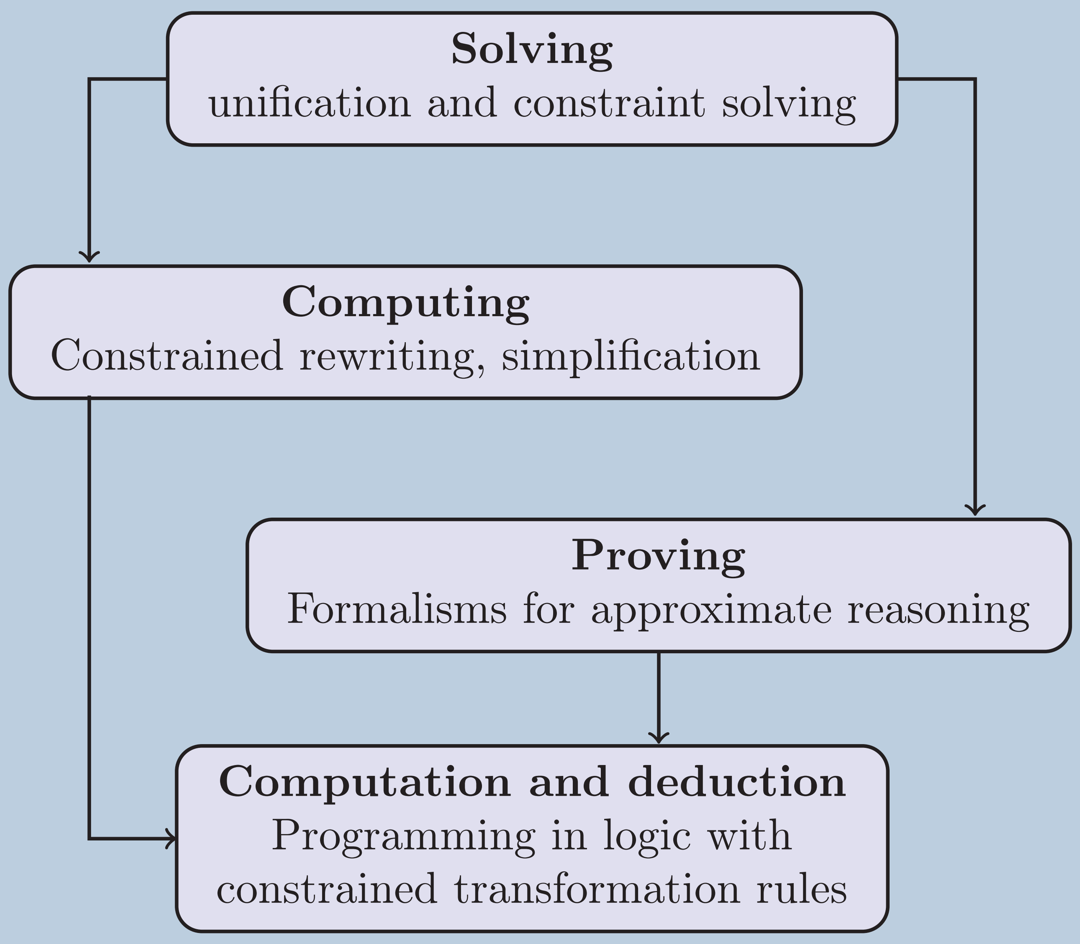
The project addresses the problem of developing novel symbolic techniques for supporting automated or semi-automated reasoning activities in theories modulo proximity and similarity relations.

Objectives



Aims

We aim at designing, analyzing, and implementing specific algorithms and procedures for each of the major reasoning activities in theories with proximity and similarity relations.



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Expected Results

- **New algorithms** and techniques for unification, matching, rewriting, and inference in fuzzy tolerance and fuzzy equivalence settings.
- These algorithms will be usually applicable to (or have analogs for) their crisp counterparts.
- **Implementation** of all these algorithms will be freely accessible from the project web page.
- Increase the international visibility of the project team members.

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