Unranked Probabilistic Theory
Project Presentation

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Introduction

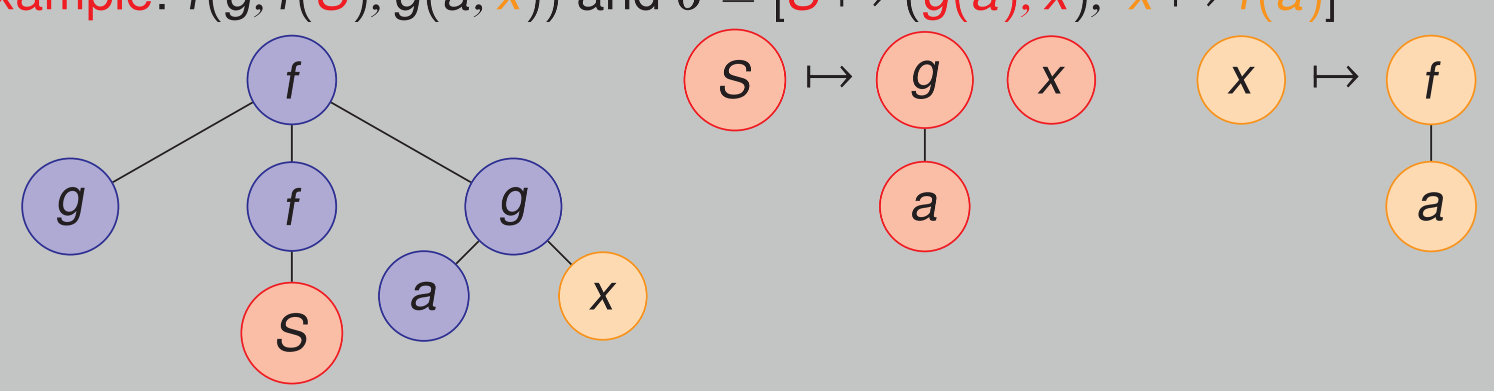
- Since the early days of Artificial Intelligence logical and probabilistic methods have been independently used to solve tasks requiring some sort of “intelligence”.
- Probability theory deals with the challenges posed by uncertainty, while logic is more often used for reasoning with perfect knowledge.
- Modern AI is area where computer science, mathematics (probability theory, numerical analysis, etc.), and logic come together.
- Researchers started combining logical and probabilistic methods in a single framework and developed several formalisms and programming tools.
- All probabilistic logic formalisms studied so far are either propositional, or permit only individual variables that can be instantiated by a single term.
- On the other hand, there are very useful theories of symbolic logic, which are using sequence variables and unranked function/predicate symbols.

Aims

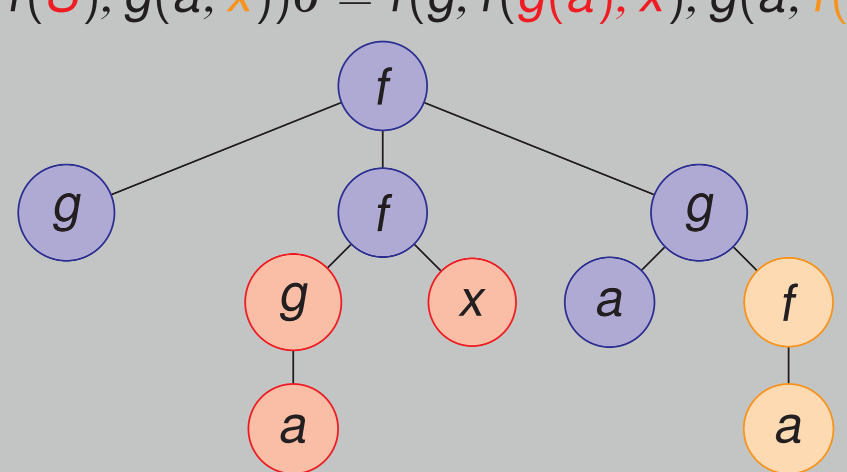
- Develop a novel theory, where sequence variables, unranked terms, and probabilistic primitives will be available together.
- Such formalism is interesting from a theoretical point of view as well as from a practical one since it provides a very flexible and expressive platform to model various problems coming from real-world applications.

Unranked Terms

- The unranked term is a first-order term, where the same function symbol can occur in different places with different numbers of arguments.
- Sequence variables can be instantiated by finite (possibly empty) sequence of (unranked) terms.
- Example: $f(g, f(S), g(a, x))$ and $\sigma = [S \mapsto (g(a), x),\ x \mapsto f(a)]$



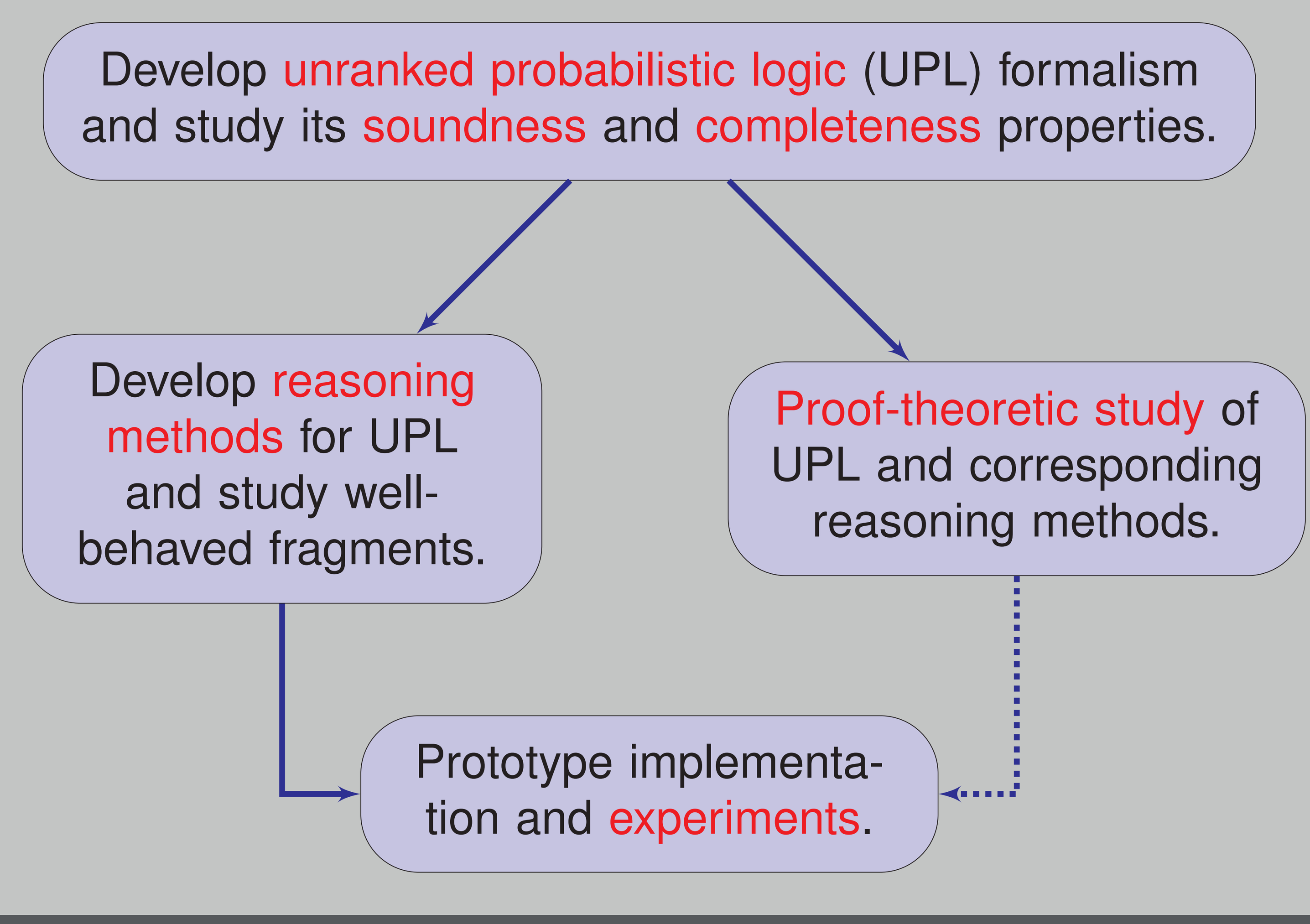
$f(g, f(S), g(a, x))\sigma = f(g, f(g(a), x), g(a, f(a)))$



Probabilistic Logics

- There are two kinds of formal probabilistic logic languages having:
  - Probabilistic operator $P_{\geq s}\alpha$: the probability of holding α is at least s.
  - Probabilistic quantifier $(Px > r)\alpha(x)$: the probability of the set $\{x \mid \alpha(x)\}$ is greater than r.
- Semantics: there is a probability that a sentence is true, but the sentence itself can be either true or false and no other value is possible (unlike many-valued logics).
- Properties: compactness usually fails and even more, the strong completeness is not available in some of probabilistic logics.

Objectives



Methods

- Following Bruno Buchberger, we identify computing, solving, and proving as three major activities in mathematical reasoning.
- Computing: provide an expression E (term, formula, program) and “evaluate” the expression for a value v, $E[x \mapsto v]$ (rewriting, simplification).
- Solving: provide an expression E with free variable x and find all values v such that $E[x \mapsto v]$ holds (unification and constraint solving).
- Proving: provide an expression E with free variable x and generate a proof (or disprove) that for all values v, $E[x \mapsto v]$ holds (reasoning methods).

Expected Results

- The expected results are both of theoretical and practical character.
- Theoretical results: development of a new, unranked probabilistic logic and the corresponding reasoning methods.
- Practical applications: expected results can be used for knowledge modelling in the areas of semantic web and web mining, medicine, activity recognition, transportation systems, e-commerce and the like.

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