Computable reduction of Σ^1_1 equivalence relations on ω

Sy Friedman and Ekaterina Fokina considered Σ^1_1 equivalence relations on ω.

Examples. Let K be a nice class of structures, and consider isomorphism of the computable members of K, identified with their indices. Similarly, consider bi-embeddability.

Fokina and Friedman defined the following pre-ordering.

Definition. Let E, E' be Σ^1_1 equivalence relations on ω. Then $E \leq_{FF} E'$ if there is a computable function f s.t. for all $m, n \in \omega$, $mEn \Leftrightarrow f(m)E'f(n)$.

A result and a question

Let $I(K)$ be the set of computable indices for elements of K, and let $E(K)$ be the set of pairs (a, b) s.t. either a, b are indices for isomorphic elements of K, or else neither is an index for an element of K.

Fokina-S. Friedman-Harizanov-K-McCoy-Montalbán. If K is one of the following, then $E(K)$ lies on top under $\leq_{FF}$: trees, torsion-free Abelian groups, p-groups.

Question. Suppose $I(K)$ is hyperarithmetical and $E(K)$ is not hyperarithmetical. Is it the case that for all Σ^1_1 equivalence relations E on ω, $E \leq_{FF} E(K)$?