# THE PERFECT TREE DICHOTOMY

For any Turing degree $\mathbf{a}$ define $\Pi(\mathbf{a})$ to be the set of Turing degrees $\mathbf{b}$ such that every $\mathbf{a}$-computable perfect tree has a member of degree $\mathbf{b}$.

For some degrees $\mathbf{a}$, $\Pi(\mathbf{a})$ is empty (if $\mathbf{a}$ is d.n.r. or PA for example).

For some degrees $\Pi(\mathbf{a})$ is the cone above $\mathbf{a}$ (this holds for measure 1 many degrees in fact).

So the question is, are these the only two possibilities?