Three open questions

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The ideal of K-trivial Turing degrees

- Each K-trivial is Turing below a c.e. K-trivial.
- Lots of characterizations of the K-trivial, but really not much known that is purely degree theoretic (not mentioning randomness).
- Pretty much all we know:
  each K-trivial is superlow, there is cone avoiding below a low c.e. degree, the low_2 c.e. upper bound, and the low Turing cover of Kučera/ Slaman (2005)

Question (Miller, N 2006)

Does the ideal $\mathbf{K}$ *of K-trivial Turing degrees have an exact pair* $\mathbf{a}, \mathbf{b}$ *in the c.e. degrees?*
That is, can we make $\mathbf{K} = [\mathbf{0}, \mathbf{a}] \cap [\mathbf{0}, \mathbf{b}]$?

Is the ideal definable without parameters in any Turing degree structure (C.e., Δ^0_2, all)?

Π^1_1 randomness

Definition

We say a set $Z \subseteq \mathbb{N}$ is Π^1_1-random if Z is in no null Π^1_1 class.

- There is a universal test, that is, a largest Π^1_1 null class $\mathcal{Q}$, by Kechris (1975); Hjorth, N 2007.
- A Π^1_1-random can be $\leq_T$ Kleenes $\mathcal{O}$ and hyperlow ($\omega_1^X = \omega_1^{CK}$ by Gandy basis theorem.
- Two characterizations via randomness enhancement known (weaker randomness notion + lowness property)

Lowness for Π^1_1 randomness

We say an oracle A is low for Π^1_1 randomness if $\mathcal{Q}^A = \mathcal{Q}$.

Question (Miller, N 2006; Hjorth/N 2007)

Is such an oracle necessarily hyperarithmetical?

We know: low for Δ^1_1 randomness = Δ^1_1 traceable (Chong, N, Yu 2008); low for Π^1_1 ML-rdness = hyperarithmetical (Hj/N 2007)

(Question discussed with Harrington, Slaman, S. Friedman and co-authors on these papers.)

We could also study Π^1_1 random hyperdegrees.

Differentiability

General direction: in n-cube $[0,1]^n$, characterize algorithmic randomness notions by differentiability of effective functions.

For instance:

Question

$\mathbf{z} \in [0,1]^n$ is computably random $\Leftrightarrow$ each computable Lipschitz function $f : [0,1]^n \to \mathbb{R}$ is differentiable at $\mathbf{z}$?

This is true for $n = 1$ by work of Freer, Kjos-Hanssen and N.