Randomness relative to an enumeration oracle

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Algorithmic randomness relative to an oracle

Definition

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1. An *$\langle A\rangle$-test* is a uniform sequence of $\Sigma^0_1\langle A\rangle$ classes $\{V_n\}_{n<\omega}$ such that $\mu V_n \leq 2^{-n}$ for every n.
2. A sequence $Z \in 2^\omega$ *passes* the test V if $Z \notin \bigcap_{n<\omega} V_n$.
3. The sequence Z is *$\langle A\rangle$-random* if it passes all $\langle A\rangle$-tests.

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Z is $\langle A \oplus \overline{A}\rangle$-random if and only if Z is A-random.

Equivalent forms

Definition

A *Solovay $\langle A\rangle$-test* is a uniform sequence of $\Sigma^0_1\langle A\rangle$ classes $\{V_n\}_{n<\omega}$ such that $\sum_n \mu V_n < \infty$.

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Definition

A *Kučera $\langle A\rangle$-test* is a $\Sigma^0_1\langle A\rangle$ class V with $\mu V < 1$.
Z *passes* a Kučera $\langle A\rangle$-test V if not every tail of Z is in V.

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Theorem

Z is $\langle A\rangle$-random if and only

- Z passes every Solovay $\langle A\rangle$-test.
- Z passes every Kučera $\langle A\rangle$-test.

Characterizing $\langle A \rangle$-randomness via (super)martingales

Recall that a function $d : 2^{<\omega} \to \mathbb{R}^{\geq 0}$ is a

1. *martingale* if for every $\sigma \in 2^{<\omega}$ we have that $d(\sigma) = \frac{d(\sigma 0)+d(\sigma 1)}{2}$;
2. *super martingale* if for every $\sigma \in 2^{<\omega}$ we have that $d(\sigma) \geq \frac{d(\sigma 0)+d(\sigma 1)}{2}$.

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$$U_d = \{(\sigma, q) \mid q \in \mathbb{Q} \ \& \ d(\sigma) > q\} \leq_e A.$$

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Z is $\langle A \rangle$-random if and only no $\langle A \rangle$-enumerable (super)martingale *succeeds* on Z.

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Comparing $\langle A\rangle$-randomness to randomness relative to total oracles

Theorem

For any set A and sequence Z, consider the following "relative randomness" notions:

1. Z is X-random for some X such that $A \leq_e X \oplus \overline{X}$ (*upwards* $\langle A\rangle$*-random*),
2. Z is $\langle A\rangle$-random,
3. Z is X-random for every X such that $X \oplus \overline{X} \leq_e A$ (*downwards* $\langle A\rangle$*-random*).

Then (1) $\Rightarrow$ (2) $\Rightarrow$ (3).

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Then (1) $\Rightarrow$ (2) $\Rightarrow$ (3).
Furthermore, each of these implications can be strict.

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Proposition

The following two conditions are equivalent:

1. A is low for randomness.
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Every 1-generic set is low for randomness.

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Proposition

Every 1-generic set is low for randomness.

Every semi-recursive set is low for randomness.

$\langle A \rangle$-randomness cannot be expressed through total oracles

Proposition

If A is weakly 1-generic relative to Z and $A \leq_e X \oplus \overline{X}$ then Z is not X-random.

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If A is weakly 1-generic relative to Z and $A \leq_e X \oplus \overline{X}$ then Z is not X-random.

Fix a weakly 2-generic set A.

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Proposition

If A is weakly 1-generic relative to Z and $A \leq_e X \oplus \overline{X}$ then Z is not X-random.

Fix a weakly 2-generic set A.

- A is weakly 1-generic relative to Chaitin's Ω, so Ω is not random relative to any total oracle above A.

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Proposition

If $X \leq_e A$ then X is not $\langle A\rangle$-random.

Fix A such that A is 1-random and of A is of quasiminimal degree.

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- A is random with respect to every total oracle below A.
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$\langle$self$\rangle$-PA sets

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1. There is a $\langle$self$\rangle$-PA set A.
2. If A is $\langle$self$\rangle$-PA then the set of total degrees below A is a Scott set.

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Theorem

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3. Every countable Scott set can be realized as the set of total degrees below a $\langle$self$\rangle$-PA set.
4. If X is PA above Y if and only if then there is a $\langle$self$\rangle$-PA A such that $Y \oplus \overline{Y} <_e A <_e X \oplus \overline{X}$.

Randomness properties of $\langle \text{self} \rangle$-PA sets

Proposition

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If A is $\langle$self$\rangle$-PA then every $\Sigma^0_1\langle A \rangle$ class of measure < 1 is covered by a $\Sigma^0_1(Y)$ class of measure < 1, for some Y such that $Y \oplus \overline{Y} \leq_e A$.

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If A is $\langle$self$\rangle$-PA then Z is $\langle A\rangle$-random if an only if Z is downwards $\langle A\rangle$-random.

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If A has continuous degree, then there is a universal $\langle A\rangle$-test.

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If A has continuous degree, then Z is $\langle A \rangle$-random iff Z is upwards $\langle A \rangle$-random.

Set randomness notions

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1. A *Martin-Löf set $\langle A\rangle$-test* is a sequence $\{W_n\}_{n<\omega}$ that is uniformly enumeration reducible to A, such that for every n $\mathrm{wt}(W_n) \leq 2^{-n}$.
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Theorem

Upwards $\langle A\rangle$-randomness $\Rightarrow$
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Theorem

There are sets Z and A such that Z is not $\langle A\rangle$-random, but Z is Solovay set $\langle A\rangle$-random.

An incompressibility approach to randomness

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Z is Solovay set $\langle A\rangle$-random if and only if $K_m(Z \upharpoonright n) - n \to \infty$ for every $\langle A\rangle$-enumerable discrete measure m.

Z is ML set $\langle A\rangle$-random if and only if $K_m(Z \upharpoonright n) \geq^+ n$ for every $\langle A\rangle$-enumerable discrete measure m.

The end

Thank you!