Open questions session

Monday, April 26, 2021 4:21 PM

The skip operator

Def: $A \le_e B$ iff there is a c.e. set W s.t

$x \in A \iff \exists v\ (\langle x, v\rangle \in W\ \&\ D_v \subseteq B)\}$

We say that W is an e-operator and $A = W(B)$.

Prop. $A \le_T B \iff A \oplus \bar{A}$ is c.e in $B \iff A \oplus \bar{A} \le_e B \oplus \bar{B}$

Def Let $K_A = \bigoplus_{e \in \omega} W_e(A)$.

We define the skip of A to be the set $A^\diamond = \overline{K_A}$

The skip was introduced and investigated in a paper by Andrews, Ganchev, Kuyper, Lempp, Miller, Soskova and S.

Properties:

① $A \le_e B$ iff $A^\diamond \le_1 B^\diamond$

$\hookrightarrow$ We can define $de(A)^\diamond = de(A^\diamond)$

②. For every $A \geq 0^\diamond$ there is some B s.t $B^\diamond \equiv_e A$.

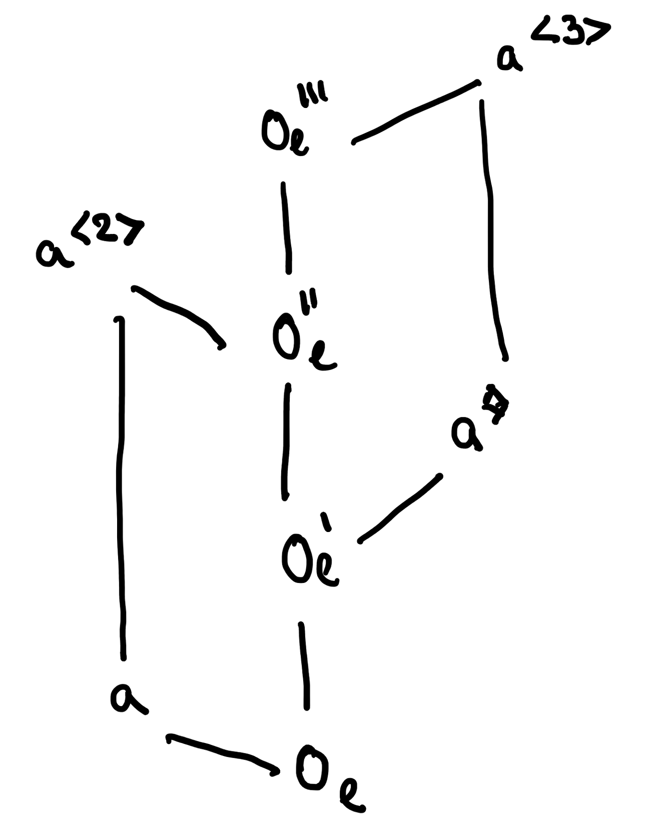
③. It is not always the case that $A \le_e A^\diamond$!

In fact, $A \le_e A^\diamond$ iff A is of cototal degree.

The e-jump of a set A is defined to be $A' = A \oplus A^\diamond$.

So for cototal (and hence total) degrees a we have $a' = a^\diamond$.

④ $A \le_e (A^\diamond)^\diamond$.



Example.

① If G is arithmetically generic then this zig-zag behaviour persists for every n.

②. There is a set A s.t $A = A^{\diamond\diamond}$, a skip 2-cycle. Such sets must bound all hyperarithmetical sets.

Questions:

①. Is there an arithmetical degree a s.t for every n $a^{\langle n\rangle}$ is nontotal?

$\rightarrow$ solved by Jun Le Goh

~~②~~. Can the HYP degrees be characterized as the degrees bounded by all skip 2-cycles?

③. Is the skip operator first order definable in the enumeration degrees?

Solution to 2:

$\{A : A$ is a skip-2-cycle$\}$ is a Σ_1^1 class. If $X \le_e A$ then X, X^c are $\Sigma_1^1(A)$, so is X is bounded by every skip 2 cycle then X, X^c are Σ_1^1 in every member of a Σ_1^1 class and hence by Harrington, Shore, Slaman - basis theorem $\Sigma_1^1 \Rightarrow$ hyp.

The reverse just follows by the existence of a skip-2-cycle.