Definability and isolation from side in the local degree theory

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Q1. Isolation from side and definability

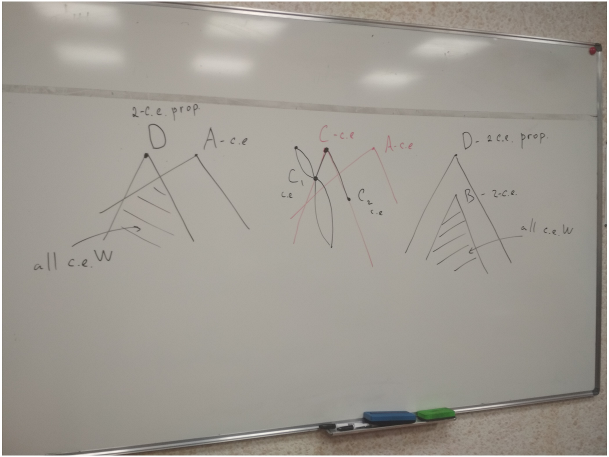
- Is any properly 2-c.e. degree either isolated or isolated from side nontrivially? *Namely, given 2-c.e. set D with a proper 2-c.e. Turing degree. Do there exist a c.e. A such that $D \not\leq_T A$ and for any c.e. $W \leq_T D$ we have $W \leq_T A$?*

- Is any properly 2-c.e. degree either isolated or pseudoisolated (by G. Wu, 2005)? *Namely, given 2-c.e. set D with a proper 2-c.e. Turing degree. Do there exist a 2-c.e. $B \leq_T D$ such that $D \not\leq_T B$ and for any c.e. $W \leq_T D$ we have $W \leq_T B$?*

- Is nontrivial isolation from side and pseudoisolation equivalent?

Note that c.e. degrees never can be nontrivially isolated from side or pseudoisolated.

Picture



Q2. Lowness and the CEA hierarchy

Theorem (Arslanov, Batyrshin, Yamaleev)

There exists noncomputable low c.e. degree $\mathbf{c}$ such that any 2-c.e. degree, which is $CEA(\mathbf{c})$, must be c.e.

Theorem (Soare, Stob, 1982)

Given noncomputable low c.e. degree $\mathbf{c}$, there exists a non-c.e. degree $\mathbf{d}$ which is $CEA(\mathbf{c})$

Question

Does the construction guarantee that the degree $CEA(\mathbf{c})$ belongs the least possible level of the Ershov hierarchy?

Question

Given low, but non-superlow, c.e. degree $\mathbf{c}$. Do there exists $CEA(\mathbf{c})$ degree which is not of 2-c.e. degree?

Q3*. The Ershov hierarchy below the halting problem

Question

Given a c.e. degree $\mathbf{c}$. When exactly it contains sets with from all proper levels of the Ershov hierarchy?

For example, superlow degrees can be only ω-c.e. On the other hand, below any high c.e. set H one can construct a set of any proper level of the Ershov hierarchy.