Wittgenstein Against Logicism

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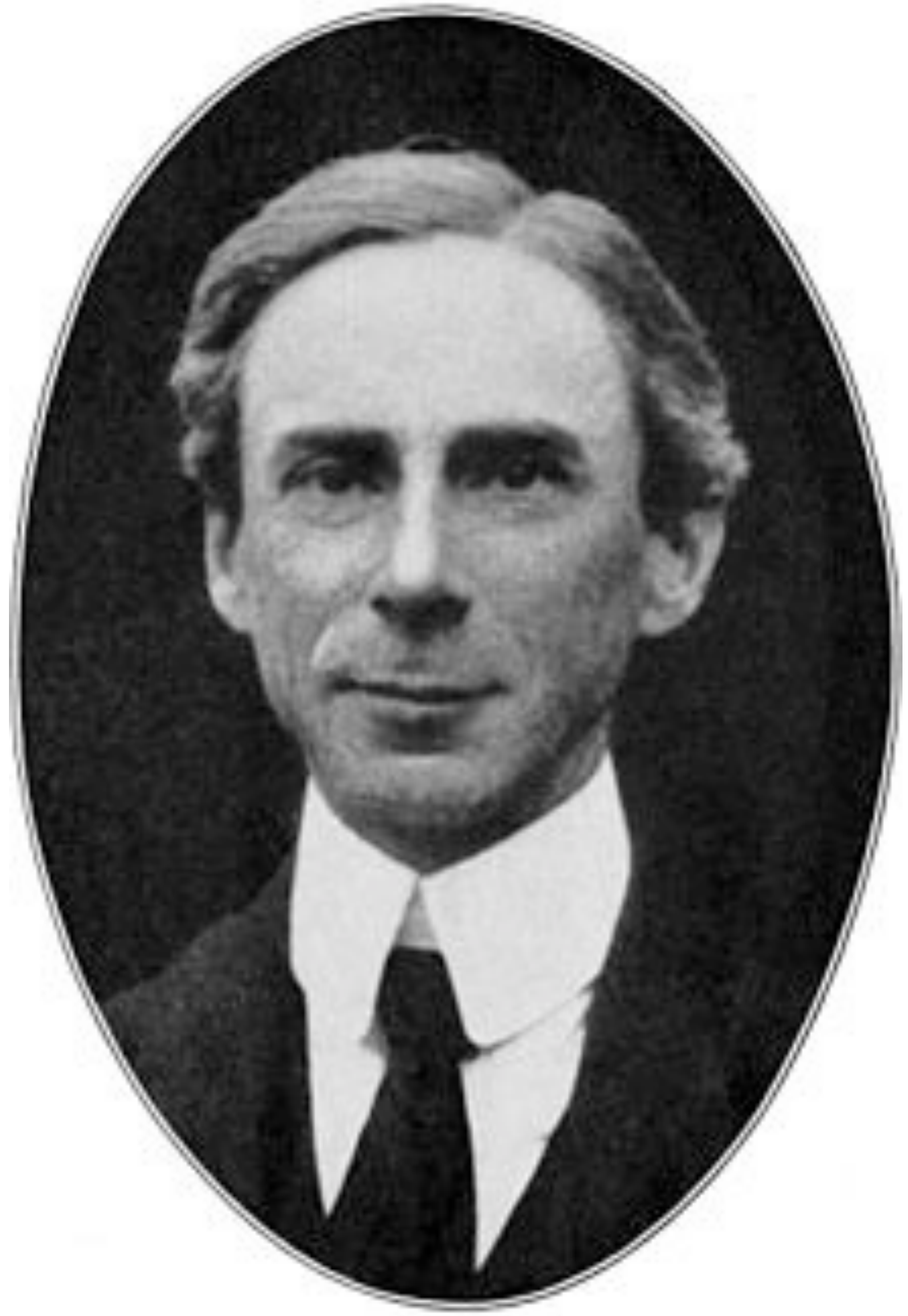
It is impossible to give a definition without enunciating a phrase, and difficult to enunciate a phrase without putting in it a name of a number, or at least the word "several" or at least a word in the plural. And then the slope is slippery, and at each instance one risks falling into a *petitio principii*.



[His] assertion appears to me to rest upon a confusion. That the types *have* an order is admitted; but it is not admitted that it is necessary to study this order as an order. ... we can make all the uses of them that are required without studying the order, just as we can distinguish a function φx from a function $\varphi(x,y)$ without knowing that the first has one argument while the second has two...So, with types, we may speak of their order in words which, strictly speaking, involve a knowledge of the ordinals, because it is obvious that we could make all the necessary uses of types without such words.

A delightful example of the way in which even mathematicians can confuse the grounds of proof with the mental or physical conditions to be satisfied if the proof is to be given in to be found E. Schröder. Under the heading "Special Axioms" he produces the following: The principle I have in mind might well be called the Axiom of Symbolic Stability. It guarantees us that throughout all our arguments and deductions the symbols remain constant in our memory — or preferably on paper.

5.5 Every truth-function is a result of successive applications to elementary propositions of the operation '(-----T)(ξ,...)'.

5.501 What the values of the variable are is something that is stipulated. The stipulations is a description of the propositions that have the variable as their representative.

We can distinguish three kinds of description: 1. direct enumeration, in which case we can simply subsitute for the variable the constants that are its values; 2. giving a function *fx* whose values for all values of *x* are the propositions to be described; 3. giving a formal law that governs the construction of the propositions, in which case the bracketed expression has as its members all the terms of a form-series.

$$x \text{ is bigger than Mars}$$

$$(\forall x){\sim}(x \text{ is bigger than Mars})$$

$$(\forall x){\sim}(x \text{ is bigger than } y)$$

$$(\forall y){\sim}(\forall x){\sim}(x \text{ is bigger than } y)$$

$$(\forall y)(\exists x)(x \text{ is bigger than } y)$$

4.1273 If we want to express in conceptual notation the general proposition, ‘*b* is a successor of *a*’, then we require an expression for the general term of the form-series

$$aRb$$

$$(\exists x):aRx.xRb$$

$$(\exists x,y):aRx.xRy.yRb$$

...

In order to express the general term of a form-series, we must use a variable, because the concept ‘term of that form-series’ is a *formal* concept. (This is what Frege and Russell overlooked: consequently the way in which they want to express general propositions like the one above is incorrect; it contains a *circulus vitiosus*.)

$m + n = p \quad$ iff $\quad$ '$aR^n b \,.\, bR^m c \supset aR^p c$' is a tautology

$(\forall x)(\forall y)(x+y=y+x)$ iff

'$\Phi(R,A,a) \supset \forall x \forall y \forall z(R^*ax \,.\, R^*ay \supset (Axyz \equiv Ayxz))$' is a tautology.

6.233 The question whether intuition [Anschauung] is needed for the solution of mathematical problems must be given the answer that here language itself provides the necessary intuition.

7 Whereof one cannot speak, thereof one must be silent.