Ramsey classes

Miodrag Sokić

Introduction
Motivation
Topological dynamics
Model theory
Ramsey theory
The KPT theory

Questions
Posets
Ultrametric spaces

Results
Posets
Ultrametric

Ramsey classes

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Motivation

Miodrag Sokić

Introduction
Motivation
Topological dynamics
Model theory
Ramsey theory
The KPT theory

Questions
Posets
Ultrametric spaces

Results
Posets
Ultrametric

- topological dynamics

Motivation

Miodrag Sokić

Introduction
Motivation
Topological dynamics
Model theory
Ramsey theory
The KPT theory

Questions
Posets
Ultrametric spaces

Results
Posets
Ultrametric

- topological dynamics
- model theory

Motivation

Miodrag Sokić
Introduction
Motivation
Topological dynamics
Model theory
Ramsey theory
The KPT theory
Questions
Posets
Ultrametric spaces
Results
Posets
Ultrametric

- topological dynamics
- model theory
- combinatorics

Motivation

Miodrag Sokić
Introduction
Motivation
Topological dynamics
Model theory
Ramsey theory
The KPT theory
Questions
Posets
Ultrametric spaces
Results
Posets
Ultrametric

- topological dynamics
- model theory
- combinatorics
- KPT

Motivation

Miodrag Sokić

Introduction
Motivation
Topological dynamics
Model theory
Ramsey theory
The KPT theory

Questions
Posets
Ultrametric spaces

Results
Posets
Ultrametric

- topological dynamics
- model theory
- combinatorics
- KPT

- group actions on compact spaces

Motivation

Miodrag Sokić

Introduction
Motivation
Topological dynamics
Model theory
Ramsey theory
The KPT theory

Questions
Posets
Ultrametric spaces

Results
Posets
Ultrametric

- topological dynamics
- model theory
- combinatorics
- KPT

- group actions on compact spaces
- Fraïssé theory

Motivation

Miodrag Sokić

Introduction
Motivation
Topological dynamics
Model theory
Ramsey theory
The KPT theory

Questions
Posets
Ultrametric spaces

Results
Posets
Ultrametric

- topological dynamics
- model theory
- combinatorics
- KPT

- group actions on compact spaces
- Fraïssé theory
- Ramsey theory

Topological dynamics

Miodrag Sokić

Introduction
Motivation
Topological dynamics
Model theory
Ramsey theory
The KPT theory

Questions
Posets
Ultrametric spaces

Results
Posets
Ultrametric

G-flow

continuous action on a compact Hausdorff space

$$G \times X \to X$$

Topological dynamics

Miodrag Sokić

Introduction
Motivation
Topological dynamics
Model theory
Ramsey theory
The KPT theory

Questions
Posets
Ultrametric spaces

Results
Posets
Ultrametric

G-flow

continuous action on a compact Hausdorff space

$$G \times X \to X$$

minimal flow

for all $x \in X$

$$\overline{G \cdot x} = X$$

Topological dynamics

G-flow

continuous action on a compact Hausdorff space

$$G \times X \to X$$

minimal flow

for all $x \in X$

$$\overline{G \cdot x} = X$$

Fact

Every G-flow contains a minimal subflow.

Topological dynamics

Miodrag Sokić

Introduction
Motivation
Topological dynamics
Model theory
Ramsey theory
The KPT theory

Questions
Posets
Ultrametric spaces

Results
Posets
Ultrametric

Theorem

Given a topological group G, there is a minimal G-flow $M(G)$ such that for any other minimal G-flow Y there exists surjective homomorphism

$$\varphi : M(G) \to Y$$
$$\varphi(gx) = g\varphi(x)$$

Moreover such G-flow $M(G)$ is uniquely determined up to isomorphism.

Topological dynamics

The minimal G-flow $M(G)$ established in the previous theorem is called the *universal minimal G-flow*.

Topological dynamics

Miodrag Sokić
Introduction
Motivation
Topological dynamics
Model theory
Ramsey theory
The KPT theory
Questions
Posets
Ultrametric spaces
Results
Posets
Ultrametric

The minimal G-flow $M(G)$ established in the previous theorem is called the *universal minimal G-flow*.
A group G is *extremely amenable*(has *fixed point on compacta property*) if $M(G) = \{*\}$.

Fraïssé classes

$\mathcal{K}$-class of finite structures in a countable signature L, closed under isomorphic images

Fraïssé class

- contains structures of arbitrary large finite cardinality

Fraïssé classes

$\mathcal{K}$-class of finite structures in a countable signature L, closed under isomorphic images

Fraïssé class

- contains structures of arbitrary large finite cardinality
- *hereditary property* (HP) $\mathbf{A} \hookrightarrow \mathbf{B} \in \mathcal{K} \implies \mathbf{A} \in \mathcal{K}$

Fraïssé classes

Miodrag Sokić
Introduction
Motivation
Topological dynamics
Model theory
Ramsey theory
The KPT theory
Questions
Posets
Ultrametric spaces
Results
Posets
Ultrametric

$\mathcal{K}$-class of finite structures in a countable signature L, closed under isomorphic images

Fraïssé class

- contains structures of arbitrary large finite cardinality
- *hereditary property* (HP) $\mathbf{A} \hookrightarrow \mathbf{B} \in \mathcal{K} \implies \mathbf{A} \in \mathcal{K}$
- *joint embedding property* (JEP) $(\forall \mathbf{A}, \mathbf{B} \in \mathcal{K})(\exists \mathbf{C} \in \mathcal{K}) [\mathbf{A} \hookrightarrow \mathbf{C} \ \& \ \mathbf{B} \hookrightarrow \mathbf{C}]$

Fraïssé classes

$\mathcal{K}$-class of finite structures in a countable signature L, closed under isomorphic images

Fraïssé class

- contains structures of arbitrary large finite cardinality
- *hereditary property* (HP) $\mathbf{A} \hookrightarrow \mathbf{B} \in \mathcal{K} \implies \mathbf{A} \in \mathcal{K}$
- *joint embedding property* (JEP) $(\forall \mathbf{A}, \mathbf{B} \in \mathcal{K})(\exists \mathbf{C} \in \mathcal{K})$ $[\mathbf{A} \hookrightarrow \mathbf{C} \;\&\; \mathbf{B} \hookrightarrow \mathbf{C}]$
- *amalgamation property* (AP) $(\forall \mathbf{A}, \mathbf{B}, \mathbf{C} \in \mathcal{K})(\forall f : \mathbf{A} \to \mathbf{B}, g : \mathbf{A} \to \mathbf{C})(\exists \mathbf{D} \in \mathcal{K})(\exists f' : \mathbf{B} \to \mathbf{D}, g' : \mathbf{C} \to \mathbf{D})[f' \circ f = g' \circ g]$

Fraïssé structures

Fraïssé structure (countable signature L)

- infinite countable

$Age(\mathbf{A})$-(age)class of finite structures embedable into $\mathbf{A}$

Fraïssé structures

Miodrag Sokić

Introduction
Motivation
Topological dynamics
Model theory
Ramsey theory
The KPT theory

Questions
Posets
Ultrametric spaces

Results
Posets
Ultrametric

Fraïssé structure (countable signature L)

- infinite countable
- *locally finite*-every finitely generated substructure is finite

$Age(\mathbf{A})$-(*age*)class of finite structures embedable into $\mathbf{A}$

Fraïssé structures

Fraïssé structure (countable signature L)

- infinite countable
- *locally finite*-every finitely generated substructure is finite
- *ultrahomogeneous*-any isomorphism between two finite substructures can be extended to (global) automorphism

$Age(\mathbf{A})$-(*age*)class of finite structures embedable into $\mathbf{A}$

"1-1"

Theorem

Let $\mathcal{K}$ be a Fraïssé class. Then there is a unique, up to isomorphism, Fraïssé structure $\mathbf{K}$ such that $\mathcal{K} = Age(\mathbf{K})$.

$$\text{Fraïssé limit-}\mathbf{K} = F\lim(\mathcal{K})$$

"1-1"

Miodrag Sokić

Introduction
Motivation
Topological dynamics
Model theory
Ramsey theory
The KPT theory

Questions
Posets
Ultrametric spaces

Results
Posets
Ultrametric

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Theorem

Let $\mathbf{K}$ be a Fraïssé structure. Then $Age(\mathbf{K})$ is a Fraïssé class.

"1-1"

Miodrag Sokić
Introduction
Motivation
Topological dynamics
Model theory
Ramsey theory
The KPT theory
Questions
Posets
Ultrametric spaces
Results
Posets
Ultrametric

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Fraïssé limit-$\mathbf{K} = F\lim(\mathcal{K})$

Theorem

Let $\mathbf{K}$ be a Fraïssé structure. Then $Age(\mathbf{K})$ is a Fraïssé class.

For every Fraïssé structure $\mathbf{K}$ we consider its automorphism group $Aut(\mathbf{K})$ with pointwise convergence topology,it is a Polish group.

Arrow notation

Miodrag Sokić
Introduction
Motivation
Topological dynamics
Model theory
Ramsey theory
The KPT theory
Questions
Posets
Ultrametric spaces
Results
Posets
Ultrametric

$\mathcal{K}$-class of finite structures in signature L

Arrow notation

Miodrag Sokić

Introduction
Motivation
Topological dynamics
Model theory
Ramsey theory
The KPT theory

Questions
Posets
Ultrametric spaces

Results
Posets
Ultrametric

$\mathcal{K}$-class of finite structures in signature L

For $\mathbf{A}, \mathbf{B} \in \mathcal{K}$ we write:

$$\binom{\mathbf{B}}{\mathbf{A}} = \{\mathbf{C} \leq \mathbf{B} : \mathbf{C} \approxeq \mathbf{A}\}.$$

Arrow notation

$\mathcal{K}$-class of finite structures in signature L

For $\mathbf{A}, \mathbf{B} \in \mathcal{K}$ we write:

$$\binom{\mathbf{B}}{\mathbf{A}} = \{\mathbf{C} \leq \mathbf{B} : \mathbf{C} \cong \mathbf{A}\}.$$

Let $\mathbf{A}, \mathbf{B}, \mathbf{C} \in \mathcal{K}$. If for any coloring

$$c : \binom{\mathbf{C}}{\mathbf{A}} \to \{1, .., r\},$$

there exist a structure $\mathbf{B}' \in \binom{\mathbf{C}}{\mathbf{B}}$ such that

$$c \restriction \binom{\mathbf{B}'}{\mathbf{A}} = const$$

then we write

$$\mathbf{C} \to (\mathbf{B})^{\mathbf{A}}_r.$$

Ramsey degree

Let $\mathcal{K}$ be a class of finite structures. If for natural numbers r and t and structures $\mathbb{A}$, $\mathbb{B}$ and $\mathbb{C}$ from $\mathcal{K}$ we have that for every coloring

$$c : \binom{\mathbb{C}}{\mathbb{A}} \rightarrow \{1, ..., r\},$$

there is $\mathbb{B}' \in \binom{\mathbb{C}}{\mathbb{B}}$ such that $c \restriction \binom{\mathbb{B}'}{\mathbb{A}}$ takes at most t many values then we write

$$\mathbb{C} \rightarrow (\mathbb{B})^{\mathbb{A}}_{r,t}.$$

We say that $\mathbb{A} \in \mathcal{K}$ has the *Ramsey degree* t if for every $\mathbb{B} \in \mathcal{K}$ and every natural number r there is $\mathbb{C} \in \mathcal{K}$ such that $\mathbb{C} \rightarrow (\mathbb{B})^{\mathbb{A}}_{r,t}$ and t is the smallest such number. We denote such a number by $t_{\mathcal{K}}(\mathbb{A})$.
Note that $\mathcal{K}$ is a Ramsey class iff $t_{\mathcal{K}}(\mathbb{A}) = 1$ for all $\mathbb{A} \in \mathcal{K}$.

(Kechris-Pestov-Todorčević 2005)

Theorem

For ordered classes

$$RP \longleftrightarrow \textit{extreme amenability},$$

$$(RP \ \textit{for}\ \mathcal{K}) \longleftrightarrow Aut(F\lim(\mathcal{K})).$$

(Kechris-Pestov-Todorčević 2005)

Theorem

For ordered classes

$$RP \longleftrightarrow \textit{extreme amenability},$$
$$(RP \textit{ for } \mathcal{K}) \longleftrightarrow Aut(F\lim(\mathcal{K})).$$

Theorem

$$(RP + OP) \longleftrightarrow \textit{universal minimal flow},$$
$$(RP \textit{ for } \mathcal{K} + OP \textit{ for } \mathcal{OK}) \longleftrightarrow$$
$$\textit{universal minimal flow } Aut(F\lim(\mathcal{K})).$$

Posets

Miodrag Sokić

Introduction
Motivation
Topological dynamics
Model theory
Ramsey theory
The KPT theory

Questions
Posets
Ultrametric spaces

Results
Posets
Ultrametric

Let $\mathcal{P}$ be the class of finite posets.
If we add arbitrary linear orderings to posets:

$$\mathcal{OP} = \{(A, \sqsubseteq, \leq) : (A, \sqsubseteq) \in \mathcal{P} \;\&\; \leq \in lo(A)\}.$$

If we add only linear extensions of partial orderings:

$$\mathcal{EP} = \{(A, \sqsubseteq, \leq) \in \mathcal{OP} : \leq \in le(\sqsubseteq)\}.$$

Limits

Miodrag Sokić
Introduction
Motivation
Topological dynamics
Model theory
Ramsey theory
The KPT theory
Questions
Posets
Ultrametric spaces
Results
Posets
Ultrametric

Fraïssé limits of posets:

- Random poset $\mathbb{P} = F\lim(\mathcal{P})$,
- Random poset with linear extension $\mathbb{EP} = F\lim(\mathcal{EP})$,
- Random poset with random ordering $\mathbb{OP} = F\lim(\mathcal{OP})$.

Problems

- UMF for $Aut(\mathbb{P})$ is known.
- $Aut(\mathbb{EP})$ is extremely amenable.
- Ramsey degree for elements in $\mathcal{P}$ is known.
- $\mathcal{EP}$ is a Ramsey class.
- $\mathcal{OP}$ is not a Ramsey class.

Problems

Miodrag Sokić

Introduction
Motivation
Topological dynamics
Model theory
Ramsey theory
The KPT theory

Questions
Posets
Ultrametric spaces

Results
Posets
Ultrametric

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- Calculate UMF for $Aut(\mathbb{OP})$.

Problems

Miodrag Sokić

Introduction
Motivation
Topological dynamics
Model theory
Ramsey theory
The KPT theory

Questions
Posets
Ultrametric spaces

Results
Posets
Ultrametric

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- $\mathcal{OP}$ is not a Ramsey class.

- Calculate UMF for $Aut(\mathbb{OP})$.
- Calculate Ramsey degrees for objects in $\mathcal{OP}$.

Ultrametric

A metric d on a set X is an *ultrametric* if for all $x, y, z \in X$ we have

$$d(x,z) \leq \max\{d(x,y), d(y,z)\}.$$

The *spectrum of a metric space* $\mathbb{X} = (X, d)$ is the set

$$Spec(\mathbb{X}) = \{d(x,y) : x \in X, y \in X, x \neq y\}.$$

We denote the class of finite ultrametric spaces by $\mathcal{U}$, and for $S \subseteq (0, +\infty)$, we consider the class

$$\mathcal{U}_S = \{\mathbb{X} \in \mathcal{U} : Spec(\mathbb{X}) \subseteq S\}.$$

Ordered spaces

Miodrag Sokić

Introduction
Motivation
Topological dynamics
Model theory
Ramsey theory
The KPT theory

Questions
Posets
Ultrametric spaces

Results
Posets
Ultrametric

Linearly ordered ultrametric spaces with distances in the set S :

$$\mathcal{OU}_S = \{(X, d, \leq) : (X, d) \in \mathcal{U}_S \ \& \ \leq \in lo(X)\}.$$

Ordered spaces

Miodrag Sokić
Introduction
Motivation
Topological dynamics
Model theory
Ramsey theory
The KPT theory
Questions
Posets
Ultrametric spaces
Results
Posets
Ultrametric

Linearly ordered ultrametric spaces with distances in the set S :

$$\mathcal{OU}_S = \{(X,d,\leq\,) : (X,d) \in \mathcal{U}_S \;\&\; \leq \in lo(X)\}.$$

If $(X,d,\leq\,) \in \mathcal{OU}_S$ and for all $x,y,z \in X$ we have

$$x \leq y \leq z \Longrightarrow d(x,y) \leq d(x,z),$$

then we say that the linear ordering $\leq$ is *convex* on the ultrametric space (X,d).

Ordered spaces

Miodrag Sokić

Introduction
Motivation
Topological dynamics
Model theory
Ramsey theory
The KPT theory

Questions
Posets
Ultrametric spaces

Results
Posets
Ultrametric

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$$x \leq y \leq z \Longrightarrow d(x, y) \leq d(x, z),$$

then we say that the linear ordering $\leq$ is *convex* on the ultrametric space (X, d).
In the convex linear ordering every ball is an interval with respect to the linear ordering.

Ordered spaces

Miodrag Sokić
Introduction
Motivation
Topological dynamics
Model theory
Ramsey theory
The KPT theory
Questions
Posets
Ultrametric spaces
Results
Posets
Ultrametric

Linearly ordered ultrametric spaces with distances in the set S :

$$\mathcal{OU}_S = \{(X, d, \leq) : (X, d) \in \mathcal{U}_S \ \& \ \leq \in lo(X)\}.$$

If $(X, d, \leq) \in \mathcal{OU}_S$ and for all $x, y, z \in X$ we have

$$x \leq y \leq z \Longrightarrow d(x, y) \leq d(x, z),$$

then we say that the linear ordering $\leq$ is *convex* on the ultrametric space (X, d).
In the convex linear ordering every ball is an interval with respect to the linear ordering.
The class of all convex linear orderings of an ultrametric space $\mathbb{X} = (X, d)$ we denote by $co(\mathbb{X})$, and we consider:

$$\mathcal{CU}_S = \{(X, d, \leq) \in \mathcal{OU}_S : \ \leq \in co((X, d))\}$$

Limits

Miodrag Sokić

Introduction
Motivation
Topological dynamics
Model theory
Ramsey theory
The KPT theory

Questions
Posets
Ultrametric spaces

Results
Posets
Ultrametric

Fraïssé limits of ultrametric spaces when S is countable:

- Random ultrametric space with spectrum$S$: $\mathbb{U}_S = F\lim(\mathcal{U}_S)$.
- Random convexly ordered ultrametric space with spectrum S: $\mathbb{CU}_S = F\lim(\mathcal{CU}_S)$.
- Random ordered ultrametric space with spectrum S: $\mathbb{OU}_S = F\lim(\mathcal{OU}_S)$.

Problems

Miodrag Sokić
Introduction
Motivation
Topological dynamics
Model theory
Ramsey theory
The KPT theory
Questions
Posets
Ultrametric spaces
Results
Posets
Ultrametric

- UMF for $Aut(\mathbb{U}_S)$ is known.
- $Aut(\mathbb{CU}_S)$ is extremely amenable.
- Ramsey degree for elements in $\mathcal{U}_S$ is known.
- $\mathcal{CU}_S$ is a Ramsey class.
- $\mathcal{OU}_S$ is not a Ramsey class.

Problems

Miodrag Sokić

Introduction
Motivation
Topological dynamics
Model theory
Ramsey theory
The KPT theory

Questions
Posets
Ultrametric spaces

Results
Posets
Ultrametric

- UMF for $Aut(\mathbb{U}_S)$ is known.
- $Aut(\mathbb{CU}_S)$ is extremely amenable.
- Ramsey degree for elements in $\mathcal{U}_S$ is known.
- $\mathcal{CU}_S$ is a Ramsey class.
- $\mathcal{OU}_S$ is not a Ramsey class.

- Calculate UMF for $Aut(\mathbb{OU}_S)$.

Problems

- UMF for $Aut(\mathbb{U}_S)$ is known.
- $Aut(\mathbb{C}\mathbb{U}_S)$ is extremely amenable.
- Ramsey degree for elements in $\mathcal{U}_S$ is known.
- $\mathcal{C}\mathcal{U}_S$ is a Ramsey class.
- $\mathcal{O}\mathcal{U}_S$ is not a Ramsey class.

- Calculate UMF for $Aut(\mathbb{O}\mathbb{U}_S)$.
- Calculate Ramsey degrees for objects in $\mathcal{O}\mathcal{U}_S$.

Expansion

Miodrag Sokić

Introduction
Motivation
Topological dynamics
Model theory
Ramsey theory
The KPT theory

Questions
Posets
Ultrametric spaces

Results
Posets
Ultrametric

We consider posets with two linear orderings:

$$\mathcal{EOP} = \{(A, \sqsubseteq, \leq, \preceq) : (A, \sqsubseteq, \leq) \in \mathcal{OP} \ \& \ (A, \sqsubseteq, \preceq) \in \mathcal{EP}\}.$$

RP

Theorem

$\mathcal{EOP}$ is a Ramsey class which satisfies OP with respect to $\mathcal{OP}$.

RP

Miodrag Sokić

Introduction
Motivation
Topological dynamics
Model theory
Ramsey theory
The KPT theory

Questions
Posets
Ultrametric spaces

Results
Posets
Ultrametric

Theorem

$\mathcal{EOP}$ *is a Ramsey class which satisfies OP with respect to* $\mathcal{OP}$.

Corollary

$$t_{\mathcal{OP}}(\mathbb{A}) = |le(\sqsubseteq)|.$$

RP

Miodrag Sokić

Introduction
Motivation
Topological dynamics
Model theory
Ramsey theory
The KPT theory

Questions
Posets
Ultrametric spaces

Results
Posets
Ultrametric

Theorem

$\mathcal{EOP}$ is a Ramsey class which satisfies OP with respect to $\mathcal{OP}$.

Corollary

$t_{\mathcal{OP}}(\mathbb{A}) = |le(\sqsubseteq)|$.

Corollary

UMF of $Aut(\mathbb{OP})$ is a closed subset of the compact space of all linear orderings $LO \subset 2^{\mathbb{N}\times\mathbb{N}}$.

RP

Miodrag Sokić

Introduction
Motivation
Topological dynamics
Model theory
Ramsey theory
The KPT theory

Questions
Posets
Ultrametric spaces

Results
Posets
Ultrametric

Theorem

$\mathcal{EOP}$ is a Ramsey class which satisfies OP with respect to $\mathcal{OP}$.

Corollary

$t_{\mathcal{OP}}(\mathbb{A}) = |le(\sqsubseteq)|.$

Corollary

UMF of $Aut(\mathbb{OP})$ is a closed subset of the compact space of all linear orderings $LO \subset 2^{\mathbb{N}\times\mathbb{N}}$.

Expansions

Miodrag Sokić

Introduction
Motivation
Topological dynamics
Model theory
Ramsey theory
The KPT theory

Questions
Posets
Ultrametric spaces

Results
Posets
Ultrametric

Ultrametric spaces with two orderings:

$$\mathcal{COU}_S = \{(X, d, \leq, \preceq) : (X, d, \leq) \in \mathcal{OU}_S \;\&\; (X, d, \preceq) \in \mathcal{CU}_S\}.$$

Expansions

Miodrag Sokić

Introduction
Motivation
Topological dynamics
Model theory
Ramsey theory
The KPT theory

Questions
Posets
Ultrametric spaces

Results
Posets
Ultrametric

Ultrametric spaces with two orderings:

$$\mathcal{COU}_S = \{(X,d,\leq,\preceq) : (X,d,\leq) \in \mathcal{OU}_S \ \& \ (X,d,\preceq) \in \mathcal{CU}_S\}.$$

In the case that the set $S \subseteq (0,+\infty)$ has a minimum $v = \min(S)$, we consider the class:

$$\mathcal{COU}_S^{\min} = \{(X,d,\leq,\preceq) \in \mathcal{COU}_S : (\forall x \in X) \leq\restriction B_v[x] = \preceq\restriction B_v[$$

Expansions

Miodrag Sokić

Introduction
Motivation
Topological dynamics
Model theory
Ramsey theory
The KPT theory

Questions
Posets
Ultrametric spaces

Results
Posets
Ultrametric

Ultrametric spaces with two orderings:

$$\mathcal{COU}_S = \{(X, d, \leq, \preceq) : (X, d, \leq) \in \mathcal{OU}_S \ \& \ (X, d, \preceq) \in \mathcal{CU}_S\}.$$

In the case that the set $S \subseteq (0, +\infty)$ has a minimum $v = \min(S)$, we consider the class:

$$\mathcal{COU}_S^{\min} = \{(X, d, \leq, \preceq) \in \mathcal{COU}_S : (\forall x \in X) \leq \restriction B_v[x] = \preceq \restriction B_v[$$

RP

Miodrag Sokić

Introduction
Motivation
Topological dynamics
Model theory
Ramsey theory
The KPT theory

Questions
Posets
Ultrametric spaces

Results
Posets
Ultrametric

Theorem

- *For non empty $S \subseteq (0, +\infty)$, $\mathcal{COU}_S$ is a Ramsey class.*

RP

Theorem

- *For non empty* $S \subseteq (0, +\infty)$, $\mathcal{COU}_S$ *is a Ramsey class.*
- *For non empty* $S \subseteq (0, +\infty)$ *where* S *has a minimum,* $\mathcal{COU}_S^{\min}$ *is a Ramsey class.*

RP

Miodrag Sokić

Introduction
Motivation
Topological dynamics
Model theory
Ramsey theory
The KPT theory

Questions
Posets
Ultrametric spaces

Results
Posets
Ultrametric

Theorem

- *For non empty* $S \subseteq (0, +\infty)$, $\mathcal{COU}_S$ *is a Ramsey class.*
- *For non empty* $S \subseteq (0, +\infty)$ *where* S *has a minimum,* $\mathcal{COU}_S^{\min}$ *is a Ramsey class.*

RP

Miodrag Sokić

Introduction
Motivation
Topological dynamics
Model theory
Ramsey theory
The KPT theory

Questions
Posets
Ultrametric spaces

Results
Posets
Ultrametric

Theorem

- *For non empty* $S \subseteq (0, +\infty)$, $\mathcal{COU}_S$ *is a Ramsey class.*
- *For non empty* $S \subseteq (0, +\infty)$ *where* S *has a minimum,* $\mathcal{COU}_S^{\min}$ *is a Ramsey class.*

Theorem

- *Let* $0 \notin S \subseteq (0, +\infty)$ *be such that* $\inf(S) \notin S$. *Then the class* $\mathcal{COU}_S$ *satisfies the ordering property with respect to* $\mathcal{OU}_S$.

RP

Miodrag Sokić

Introduction
Motivation
Topological dynamics
Model theory
Ramsey theory
The KPT theory

Questions
Posets
Ultrametric spaces

Results
Posets
Ultrametric

Theorem

- *For non empty $S \subseteq (0, +\infty)$, $\mathcal{COU}_S$ is a Ramsey class.*
- *For non empty $S \subseteq (0, +\infty)$ where S has a minimum, $\mathcal{COU}_S^{\min}$ is a Ramsey class.*

Theorem

- *Let $0 \notin S \subseteq (0, +\infty)$ be such that $\inf(S) \notin S$. Then the class $\mathcal{COU}_S$ satisfies the ordering property with respect to $\mathcal{OU}_S$.*
- *Let $0 \notin S \subseteq (0, +\infty)$ be such that $\inf(S) \in S$. Then the class $\mathcal{COU}_S^{\min}$ satisfies the ordering property with respect to $\mathcal{OU}_S$.*

RP

Miodrag Sokić

Introduction
Motivation
Topological dynamics
Model theory
Ramsey theory
The KPT theory

Questions
Posets
Ultrametric spaces

Results
Posets
Ultrametric

Theorem

- *For non empty* $S \subseteq (0, +\infty)$, $\mathcal{COU}_S$ *is a Ramsey class.*
- *For non empty* $S \subseteq (0, +\infty)$ *where* S *has a minimum,* $\mathcal{COU}_S^{\min}$ *is a Ramsey class.*

Theorem

- *Let* $0 \notin S \subseteq (0, +\infty)$ *be such that* $\inf(S) \notin S$. *Then the class* $\mathcal{COU}_S$ *satisfies the ordering property with respect to* $\mathcal{OU}_S$.
- *Let* $0 \notin S \subseteq (0, +\infty)$ *be such that* $\inf(S) \in S$. *Then the class* $\mathcal{COU}_S^{\min}$ *satisfies the ordering property with respect to* $\mathcal{OU}_S$.

RP

Miodrag Sokić

Introduction
Motivation
Topological dynamics
Model theory
Ramsey theory
The KPT theory

Questions
Posets
Ultrametric spaces

Results
Posets
Ultrametric

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Application

Miodrag Sokić

Introduction
Motivation
Topological dynamics
Model theory
Ramsey theory
The KPT theory

Questions
Posets
Ultrametric spaces

Results
Posets
Ultrametric

Theorem

- *Let $S \subset (0, +\infty)$ be non empty without a minimal element. Then for $\mathbb{A} = (A, d, \leq) \in \mathcal{OU}_S$ we have*

$$t_{\mathcal{OU}_S}(\mathbb{A}) = |co((A, d))|.$$

Application

Miodrag Sokić
Introduction
Motivation
Topological dynamics
Model theory
Ramsey theory
The KPT theory
Questions
Posets
Ultrametric spaces
Results
Posets
Ultrametric

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Application

Miodrag Sokić

Introduction
Motivation
Topological dynamics
Model theory
Ramsey theory
The KPT theory

Questions
Posets
Ultrametric spaces

Results
Posets
Ultrametric

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Application

Miodrag Sokić
Introduction
Motivation
Topological dynamics
Model theory
Ramsey theory
The KPT theory
Questions
Posets
Ultrametric spaces
Results
Posets
Ultrametric

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Corollary

UMF of $Aut(\mathbb{OU}_S)$ *is closed subset of* LO.

Application

Miodrag Sokić

Introduction
Motivation
Topological dynamics
Model theory
Ramsey theory
The KPT theory

Questions
Posets
Ultrametric spaces

Results
Posets
Ultrametric

Theorem

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Ramsey classes

Miodrag Sokić

Introduction
Motivation
Topological dynamics
Model theory
Ramsey theory
The KPT theory

Questions
Posets
Ultrametric spaces

Results
Posets
Ultrametric

Thank you