Borel-Definable Algebraic Topology

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Partly joint work with Jeffrey Bergfalk[1] (University of Vienna) and Aristotelis Panagiotopoulos (Carnegie Mellon University)

[1] Winner of the 2021 Mary Ellen Rudin Award for applications of logic to topology

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2. Borel-definable homological algebra

3. Borel-definable algebraic topology
   - Finer invariants
   - Richer invariants
   - Rigid invariants

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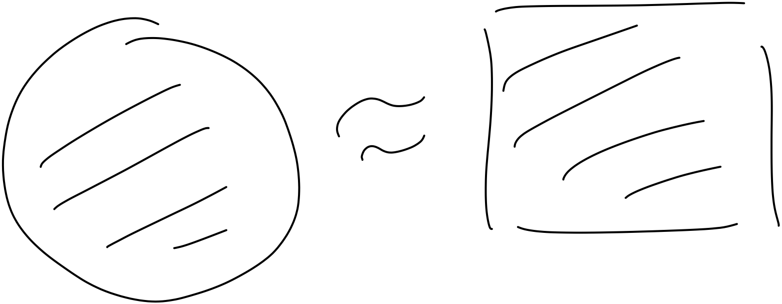
Topogy and Classification

In topology one tries to classify spaces up to homeomorphism $\approx$

In homotopy theory the relation of homotopy equivalence is considered



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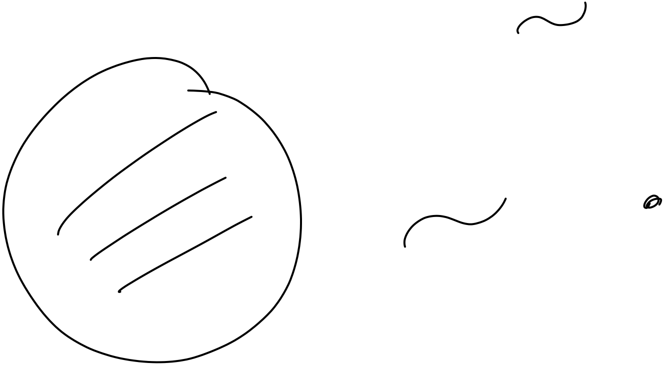

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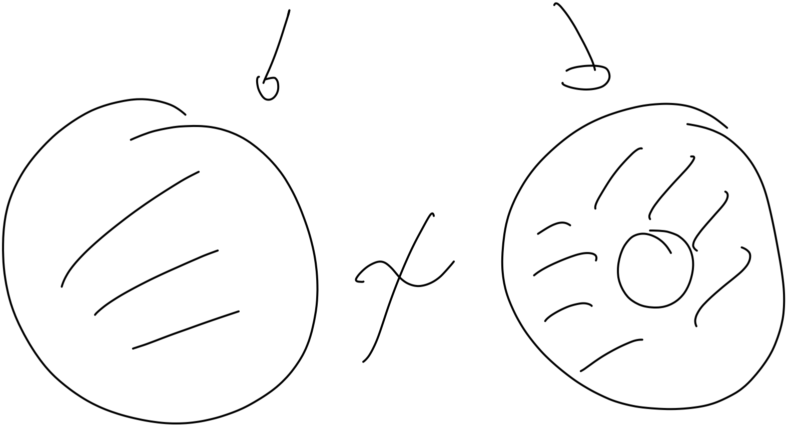

Topogy and Classification

In topology one tries to classify spaces up to homeomorphism

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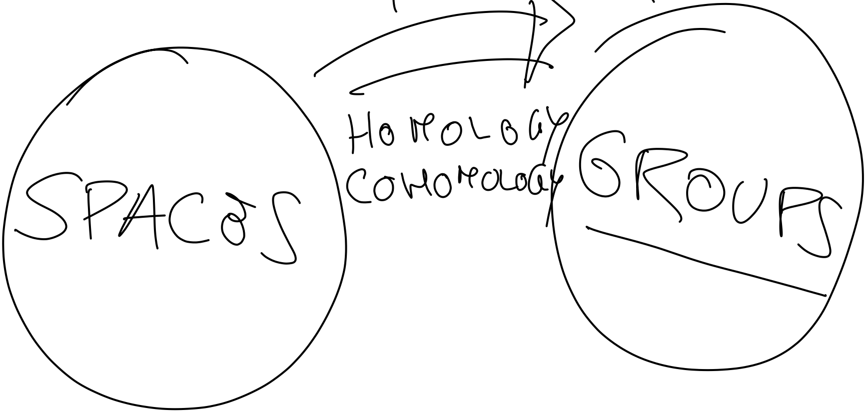
Invariants in Algebraic Topology

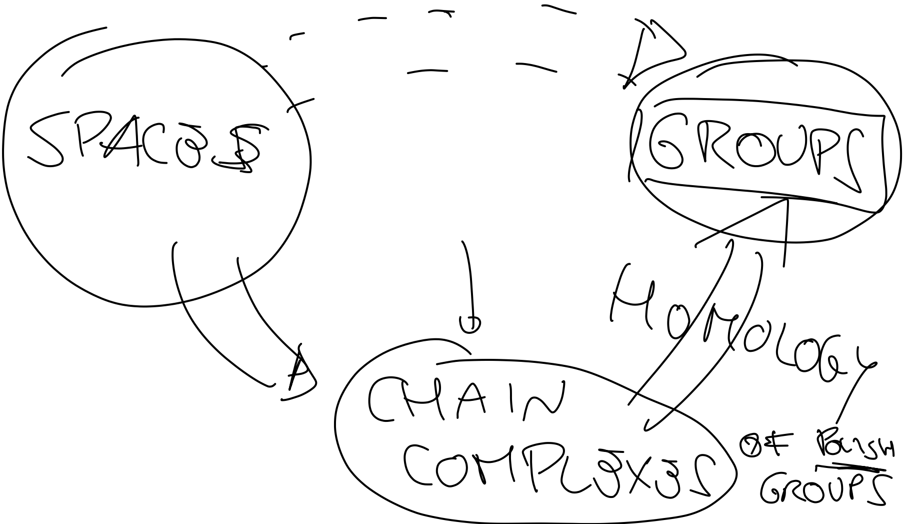
One attaches to topological spaces algebraic invariants such as groups

(All the groups will be abelian.) FUNCTOR



From chain complexes to groups

The final invariant (group) is obtained by passing via chain complexes.



Why Polish groups?

Polish: second countable, topology induced by a complete metric

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Polish: second countable, topology induced by a complete metric

The class of Polish groups:

- contains locally compact groups
- contains spaces from analysis (Banach spaces, operator algebras)
- is closed under countable products and inverse limits
- is closed under closed subgroups and quotients by closed subgroups
- the algebra of Borel sets of a Polish group is standard (isomorphic to the algebra of Borel sets of $\mathbb{R}$)

The homology of a Polish chain complex

Consider a chain complex of Polish groups A_*:

CONTINUOUS

$$\cdots \longrightarrow A_0 \xrightarrow{\varphi_0} A_1 \xrightarrow{\varphi_1} A_2 \longrightarrow \cdots$$

POLISH

POLISH

$$\boxed{H_1(A_*)} = \frac{\mathrm{Ker}(\varphi_1)}{\varphi_0(A_0)}$$

NOT CLOSED

The homology of a Polish chain complex

Consider a chain complex of Polish groups A_*:

$$\cdots \longrightarrow A_0 \xrightarrow{\varphi_0} A_1 \xrightarrow{\varphi_1} A_2 \longrightarrow \cdots$$

In A History of Algebraic and Differential Topology, Dieudonné writes of

> *a trend that was very popular until around 1950 (although later all but abandoned), namely, to consider homology groups as topological groups for suitably chosen topologies.*

The problem with cokernels

Polish group form an abelian category, but. . .

Polish groups are not an abelian subcategory of the category of groups

$$G \xrightarrow{\varphi} H \longrightarrow \frac{H}{\overline{\varphi(G)}}$$

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G. P. C.

Solution: add cokernels

Consider a category having as objects exact sequences of the form

$$0 \longrightarrow K \xrightarrow{\eta} \hat{G} \longrightarrow G \longrightarrow 0$$

where:

- K and $\hat{G}$ are Polish groups
- η is a continuous group homomorphism

POLISH

$$G = \hat{G} / N$$

POLISHABLE

$$N = \eta(\hat{G})$$

Solution: add cokernels

Consider a category having as objects exact sequences of the form

$$0 \longrightarrow K \xrightarrow{\eta} \hat{G} \longrightarrow G \longrightarrow 0$$

where:

- K and $\hat{G}$ are Polish groups
- η is a continuous group homomorphism

We call such an exact sequence a group with a Polish cover

If $N := \eta(K)$, then we can identify it with

$$G = \hat{G}/N$$

The category of groups with a Polish cover

The morphisms are the group homomorphisms that are Borel-definable, namely induced by a Borel function "upstairs"

$$\begin{tikzcd}
\hat{G} \arrow[r, "\hat{\varphi}"] \arrow[d] & \hat{H} \arrow[d] \\
\hat{G}/N \arrow[r, "\varphi"'] \arrow[ur, phantom, "\circlearrowright"] & \hat{H}/M
\end{tikzcd}$$

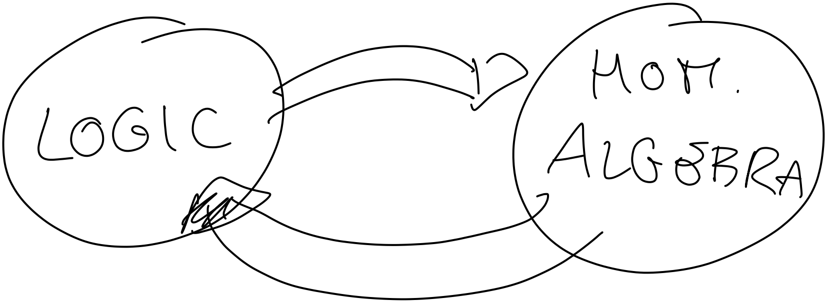
BOREL FUNCTION

GROUP HOM.

The category of groups with a Polish cover

Theorem (L., 2021)

The category of groups with Polish cover is an abelian category, which is an abelian subcategory of the category of groups.



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The category of groups with a Polish cover is the natural context to develop Borel-definable homological algebra.

Definable homological algebra

For example, the homological invariants

$$\mathrm{Ext}(A, B) \longleftarrow \text{G.P.C.}$$

$$\mathrm{Hom}(A, B) \longleftarrow \text{POLISH GROUP}$$

for countable groups A and B, are groups with a Polish cover.

$$\mathrm{Ext}(A,B) = \frac{\text{COCYCLES}}{\text{COBOUNDARIES}}$$

1942

Definable homological algebra

Theorem (Bergfalk, L., Panagiotopouos, 2019)

The definable homological invariant $\mathrm{Ext}(-,\mathbb{Z})$ *is a complete invariant for countable torsion-free groups.*

$$A_0 \simeq A_1 \Longleftrightarrow \mathrm{Ext}(A_0,\mathbb{Z})$$

BOREL DEF. $\longrightarrow$ $\wr$

$$\mathrm{Ext}(A_1,\mathbb{Z})$$

Definable homological algebra

Theorem (Bergfalk, L., Panagiotopouos, 2019)

The definable homological invariant $\mathrm{Ext}(-,\mathbb{Z})$ *is a complete invariant for countable torsion-free groups.*

DEFINABLE

In fact, $\mathrm{Ext}(-,\mathbb{Z})$ *is a fully faithful functor from countable torsion-free groups to groups with a Polish cover*

Definable homological algebra

Theorem (Bergfalk, L., Panagiotopouos, 2019)

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In fact, $\mathrm{Ext}(-, \mathbb{Z})$ *is a fully faithful functor from countable torsion-free groups to groups with a Polish cover*

This does not hold for the purely algebraic Ext.

More definable homological algebra

Project

Show that the category of groups with Polish cover is the Adelman abelianization of the category of Polish groups

More definable homological algebra

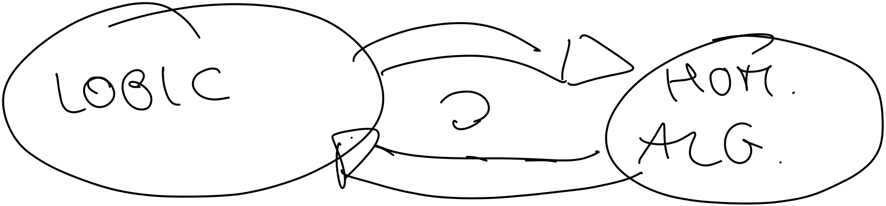
Project

Show that the category of groups with Polish cover is the *Adelman abelianization* *of the category of Polish groups*

Project

Determine injective and projective objects.

Are there enough injectives/projectives?



More definable homological algebra

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Show that the category of groups with Polish cover is the Adelman abelianization of the category of Polish groups

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Determine injective and projective objects.

Are there enough injectives/projectives?

Project

Generalize to R-modules.

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G. P. C.

Definable refinements of algebraic invariants

Virtually all group invariants from algebraic topology can be refined and seen as invariants taking values in the category of groups with Polish cover

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Virtually all group invariants from algebraic topology can be refined and seen as invariants taking values in the category of groups with Polish cover

Advantages of the definable versions:

1. finer invariants (distinguish more spaces, more powerful invariants)
2. richer invariants (e.g., one can study their Borel class and Borel rank)
3. rigid invariants (fewer automorphisms, better grasp on the dynamics)

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Finer invariants

Theorem (Bergfalk, L., Panagiotopoulos, 2018–2020)

The following invariants admit definable refinements:

- *Steenrod homology of compact spaces*
- K*-homology of compact spaces and of C*-algebras*
- *Čech cohomology of locally compact spaces*

$($

Finer invariants

Theorem (Bergfalk, L., Panagiotopoulos, 2018–2020)

The following invariants admit *definable refinements*:

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- K*-homology of compact spaces and of C*-algebras*
- *Čech cohomology of locally compact spaces*

Furthermore:

NOT

1. *~~definable~~ Steenrod homology* $H_*(-)$ *is a complete invariant for solenoids (inverse limits of tori)* FRACTALS
2. *definable* K*-homology is a complete invariant for solenoids*

NOT

3. *~~definable~~ Čech cohomology* $H^*(-)$ *is a complete invariant for mapping telescopes of tori or spheres*

CW-COMPLEXES

Solenoids

A solenoid is simply an inverse limit of copies of $\mathbb{T}$

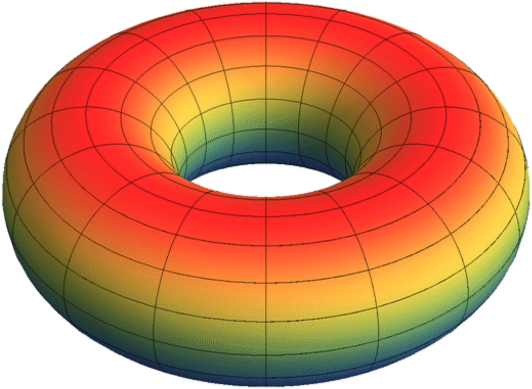
$$\mathbb{T} \xleftarrow{f_p} \mathbb{T} \xleftarrow{f_p} \mathbb{T} \xleftarrow{f_p} \cdots$$

X_p $\quad$ p-ADIC SOLENOID

Solenoids

A solenoid is simply an inverse limit of copies of $\mathbb{T}$

A concrete geometric realization in $\mathbb{R}^3$ of a solenoid can be obtained as intersection of a sequence of nested solid tori



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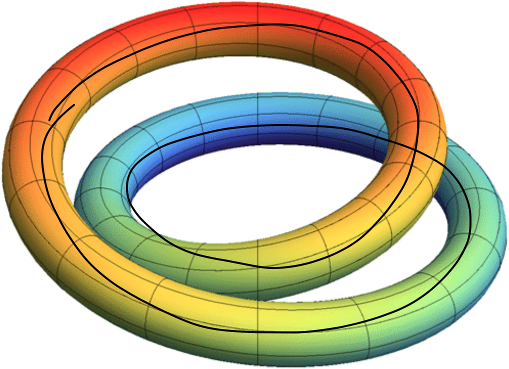

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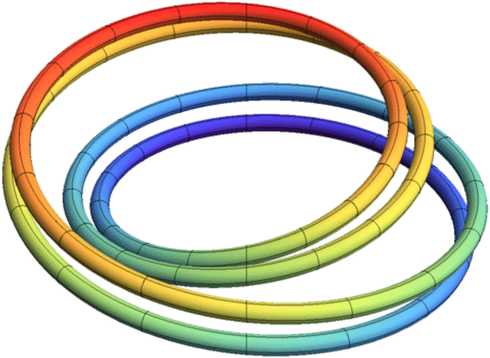

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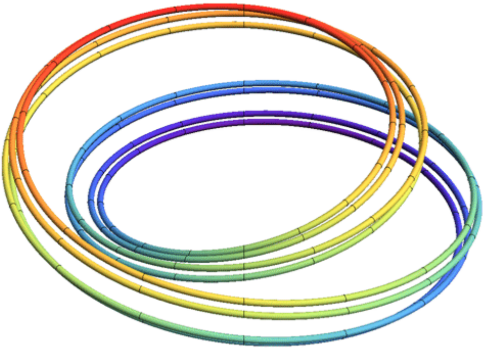

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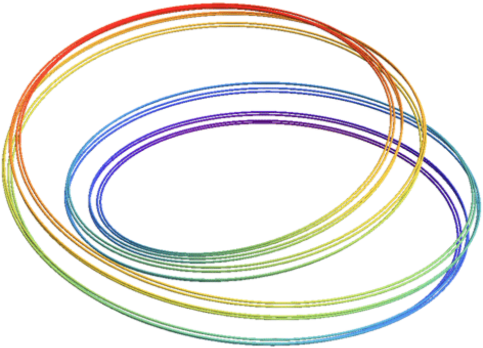

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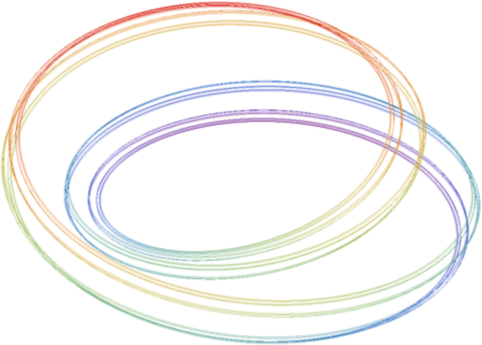
A concrete geometric realization in $\mathbb{R}^3$ of a solenoid can be obtained as intersection of a sequence of nested solid tori

X_2



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The Borel Hierarchy

Let X be a Polish space and $A \subseteq X$:

1.
   - A is $\mathbf{\Sigma}_1$ iff A is open in X
   - A is $\mathbf{\Pi}_1$ iff A is closed in X

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   - A is $\boldsymbol{\Sigma}_2$ iff A is $\underline{\text{union of } \boldsymbol{\Pi}_1}$ sets F_σ
   - A is $\boldsymbol{\Pi}_2$ iff A is intersection of $\boldsymbol{\Sigma}_1$ sets G_δ

The Borel Hierarchy

Let X be a Polish space and $A \subseteq X$:

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   - A is $\mathbf{\Pi}_1$ iff A is closed in X
2. - A is $\mathbf{\Sigma}_2$ iff A is union of $\mathbf{\Pi}_1$ sets
   - A is $\mathbf{\Pi}_2$ iff A is intersection of $\mathbf{\Sigma}_1$ sets
3. - A is $\mathbf{\Sigma}_3$ iff A is union of $\mathbf{\Pi}_2$ sets $G_{\delta\sigma}$
   - A is $\mathbf{\Pi}_3$ iff A is intersection of $\mathbf{\Sigma}_2$ sets

...

$\alpha < \omega_1$ $\quad \Sigma_\alpha \quad \Pi_\alpha$

The Borel Hierarchy

Let X be a Polish space and $A \subseteq X$:

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BOREL CLASSES

2. - A is $\mathbf{\Sigma}_2$ iff A is union of $\mathbf{\Pi}_1$ sets
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3. - A is $\mathbf{\Sigma}_3$ iff A is union of $\mathbf{\Pi}_2$ sets
   - A is $\mathbf{\Pi}_3$ iff A is intersection of $\mathbf{\Sigma}_2$ sets

...

The Borel rank of Borel set $A \subseteq X$ is the least α such that A is $\mathbf{\Pi}_\alpha$

Subobjects

Let $G = \hat{G}/N$ be a group with a Polish cover.

A subgroup H of G is Polishable if it is of the form

$$H = \hat{H}/N$$

for some Polishable BOREL subgroup $\hat{H}$ of $\hat{G}$ containing N.

$$\begin{tikzcd} N \arrow[r, phantom, "\subseteq"] & \hat{H} \arrow[d] \arrow[r, phantom, "\subseteq"] & \hat{G} \arrow[d] \\ & \hat{H}/N \arrow[r, phantom, "\subseteq"] & \hat{G}/N \end{tikzcd}$$

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A subgroup H of G is Polishable if it is of the form

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Such a subgroup H of G has a Borel class and a Borel rank.

These are by definition the Borel class and the Borel rank of $\hat{H}$ in $\hat{G}$.

BOREL

Solecki subgroups

Theorem (L., 2021, building on Solecki 1999 and Farah–Solecki 2006)

Let G be a group with a Polish cover, and let α be a countable ordinal.

There exists a smallest $\mathbf{\Pi}_{1+\alpha+1}$ Polishable subgroup $s_\alpha(G)$ of G.

$$s_2(G) \subseteq s_1(G) \subseteq s_0(G) \subseteq G$$

$$s_2(G) = \overline{\Pi_4} \qquad \overline{\Pi_3} \qquad s_0(G) = \overline{\{0\}}$$

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Remark

We have that $s_0(G)$ is the closure of $\{0\}$.

Solecki subgroups and Ulm subgroups

Theorem (L., 2021)

For every countable ordinal α, and torsion groups A and B,

$$s_\alpha\left(\mathrm{Ext}(A, B)\right)$$

LOGIC

ALGEBRA

is equal to the $(1+\alpha)$-th Ulm subgroup

$$u_{1+\alpha}(\mathrm{Ext}(A, B))$$

Solecki subgroups and Ulm subgroups

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Corollary (Eilenberg–MacLane, 1942)

The closure of $\{0\}$ in $\mathrm{Ext}\,(A, B)$ is equal to the first Ulm subgroup.

$\alpha = 0$

Borel class and Borel rank

Theorem (L., 2021)

Computation of the Borel rank of $\{0\}$ *in* $\mathrm{Ext}(A, B)$ *when* A *and* B *are either torsion or torsion-free.*

- *In the torsion case, the Borel rank can be arbitrarily large*

COMPLEXITY
OF
EXTENSIONS

Borel class and Borel rank

Theorem (L., 2021)

Computation of the Borel rank of $\{0\}$ in $\mathrm{Ext}(A, B)$ when A and B are either torsion or torsion-free.

- *In the torsion case, the Borel rank can be arbitrarily large*
- *In the torsion-free case, the Borel rank is at most 3*

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- *In the torsion case, the Borel rank can be arbitrarily large*
- *In the torsion-free case, the Borel rank is at most* 3

Project

Extend the previous result to arbitrary countable groups (or R-modules)

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Rigidity

Groups with a Polish cover are more rigid than discrete groups:
they have fewer automorphisms

The reason is that not all group automorphisms are Borel-definable

p-adic numbers

Let $\mathbb{Q}_p$ be the p-adic numbers (seen as additive locally profinite group)

We have a canonical action $\mathbb{Q}^\times \curvearrowright \mathbb{Q}_p$ by multiplication

This induces an action $\mathbb{Q}^\times \curvearrowright \mathbb{Q}_p/\mathbb{Q}$

POLISHABLE

POLISH

$$\underline{\mathbb{Q}} \subseteq \mathbb{Q}_p$$

$$\underline{\mathbb{Q}^\times \curvearrowright \mathbb{Q}_p/\mathbb{Q}}$$

Ulam stability of p-adics

Theorem (Bergfalk, L., Panagiotopoulos, 2019)

All the Borel-definable automorphisms of $\mathbb{Q}_p/\mathbb{Q}$ are given by the action

$$\mathbb{Q}^\times \curvearrowright \mathbb{Q}_p/\mathbb{Q}$$

This shows that there exist $\aleph_0$ Borel-definable automorphisms of $\mathbb{Q}_p/\mathbb{Q}$

In contrast, there exist $2^{2^{\aleph_0}}$ automorphisms of $\mathbb{Q}_p/\mathbb{Q}$

Solenoid complements

We denote by S^d the one-point compactification of $\mathbb{R}^d$

Let $X_p \subseteq \underline{S^3}$ be a geometric realization of the p-adic solenoid

Let $[S^3 \setminus X_p, S^2]$ be the space of homotopy classes of maps $S^3 \setminus X_p \to S^2$

$$X_p \subseteq S^3$$

$$S^3 \setminus X_p \to S^2$$

Some history

1936: Borsuk and Eilenberg raise the problem of understanding the space

$$[S^3 \setminus X_p, S^2]$$

COMPLICATED

Some history

1936: Borsuk and Eilenberg raise the problem of understanding the space

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1940: Eilenberg develops obstruction theory and establishes the (Borel-definable) bijection

$$[S^3 \setminus X_p, S^2] \cong H^2(S^3 \setminus X_p)$$

Some history

1940: Steenrod introduces Steenrod homology theory and proves Steenrod Duality, and in particular the (Borel-definable) isomorphism

$$H^2(S^3 \setminus X_p) \cong H_0(X_p)$$

UNCOUNTABLE

Some history

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$$H^2(S^3 \setminus X_p) \cong H_0(X_p)$$

1942: Eilenberg and MacLane prove the Universal Coefficient Theorem and the (Borel-definable) isomorphisms

$$H_0(X_p) \cong \mathrm{Ext}\left(H^1(X_p), \mathbb{Z}\right) \cong \mathrm{Ext}\left(\mathbb{Z}[1/p], \mathbb{Z}\right) \cong \mathbb{Q}_p/\mathbb{Q}$$

Some history

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Putting it all together, there is a Borel-definable bijection

$$[S^3 \setminus X_p, S^2] \cong \mathbb{Q}_p/\mathbb{Q}$$

Equivariant classification

Let $\mathcal{E}(S^3 \setminus X_p)$ be the space of homotopy automorphisms of $S^3 \setminus X_p$

There is a canonical Borel-definable action

$$[S^3 \setminus X_p, S^2] \curvearrowleft \mathcal{E}(S^3 \setminus X_p)$$

Equivariant classification

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$$[S^3 \setminus X_p, S^2] \curvearrowleft \mathcal{E}(S^3 \setminus X_p)$$

Using the rigidity of $\mathbb{Q}_p/\mathbb{Q}$ we can conclude that the action

$$[S^3 \setminus X_p, S^2] \curvearrowleft \mathcal{E}(S^3 \setminus X_p)$$

corresponds to the canonical action

$$\mathbb{Q}^\times \curvearrowright \mathbb{Q}_p/\mathbb{Q}$$

Equivariant classification

So the problem of classifying the orbits of

$$\underbrace{[S^3 \setminus X_p, S^2]}_{} \curvearrowleft \mathcal{E}(S^3 \setminus X_p)$$

is the same as the problem of classifying the orbits of

$$\mathbb{Q}^\times \curvearrowright \underline{\mathbb{Q}_p/\mathbb{Q}}$$

which in turn is the same as the problem of classifying the orbits of

$$\mathrm{Aff}(\underline{\mathbb{Q}}) = \underline{\mathbb{Q}^\times \ltimes \mathbb{Q}} \curvearrowright \underline{\mathbb{Q}_p}$$

Equivariant classification

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which in turn is the same as the problem of classifying the orbits of

$$\mathbb{Q}^\times \ltimes \mathbb{Q} \curvearrowright \mathbb{Q}_p$$

In particular, there exist $2^{\aleph_0}$ such orbits

Higher dimensions

There are higher-dimensional analogues, where

$$X_p^d \subseteq S^{d+2}$$

is the product of d copies of the p-adic solenoid.

Higher dimensions

There are higher-dimensional analogues, where

$$X_p^d \subseteq S^{d+2}$$

is the product of d copies of the p-adic solenoid.

In this case we have that the Borel-definable action

$$[S^{d+2} \setminus X_p^d, S^{d+1}] \curvearrowleft \mathcal{E}(S^{d+2} \setminus X_p^d)$$

corresponds to the action

$$\mathrm{GL}_d(\mathbb{Q}) \curvearrowright \mathbb{Q}_p^d/\mathbb{Q}^d$$

Measuring the complexity

Using tools from IOANA

- ergodic theory (superrigidity for profinite actions), and
- algebraic geometry (superrigidity for p-adic Lie groups) ← MARGOULIS

one can compare the Borel complexity of such actions.

Theorem (Bergfalk, L., Panagiotopoulos, 2019)

The Borel complexity of classifying the orbits of

$$[S^{d+2} \setminus X_p^d, S^{d+1}] \curvearrowleft \mathcal{E}(S^{d+2} \setminus X_p^d)$$

or equivalently

$$\mathrm{Aff}(\mathbb{Q}^d) = \mathrm{GL}_d(\mathbb{Q}) \ltimes \mathbb{Q}^d \curvearrowright \mathbb{Q}_p^d$$

strictly increases with d.

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or equivalently

$$\underline{\mathrm{GL}_d(\mathbb{Q}) \ltimes \mathbb{Q}^d \curvearrowright \mathbb{Q}_p^d} \qquad SL_{\cancel{d}}(\mathbb{Z})$$

strictly increases with d.

$d \geq 3$

For $d \geq 3$, these problems for different primes are incomparable from the perspective of Borel complexity.

Further directions

Project

Applications to solenoidal manifolds and other self-similar objects (fractals).

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Develop definable refinement of coarse geometry.

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Develop definable refinement of coarse geometry.

Project

Definable group cohomology.