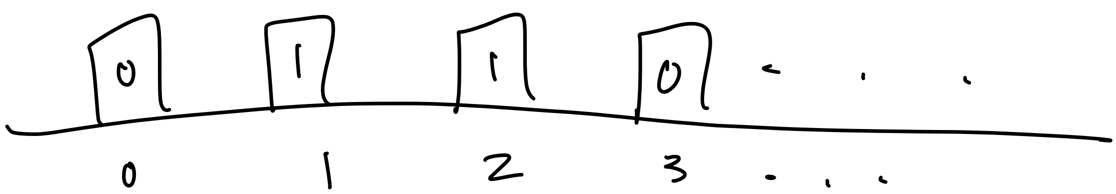
A computably small set that is not intrinsically small

Uniform MWC-stochasticity 0   Uniform KL-stoch 0

(Joint with Justin Miller)

Consider infinite & computable variant of Monty Hall Problem.

- Infinite doors in a row



- Host A hides goat / car behind each door
  (0) (1)
- Infinitely many doors hide cars
- Contestant f chooses inf many doors to open
  f(0), f(1), . . .
- Contestant behave like Turing machine $f \leq_T \emptyset$
- Contestant wins if selected doors have non-zero density of cars.

Type of contestants

- Disorderly or orderly.

  Disorderly f may choose door i after door j>i.

- Adaptive vs non-adaptive

  Non-adaptive: Next door picked does not depend on earlier outcomes

Dependency diagram for set A (for $\alpha = 0$)

(KL-stochasticity 0)

A Beat adaptive, disorderly

(MWC-stoch 0)

Beat adaptive orderly

Beat non-adapt disorderly (intrinsically small) (Unif KL-stoch 0)

[K-M]

Beat non-adapt orderly (computably small)

(Unif MWC-stoch 0)

Open Q Does $\not\uparrow$ also hold for degrees?

Remarks Other variations of this setup have been considered

- Allow contestants to bet diff \$ on different doors (Martingale)
- Host declares they have α density of cars. Contestant wins if selected doors give different density.

(intrinsic density α computable — α)

Lemma (Justin) Diagram holds also for $\alpha > 0$.

<u>Thm</u> [Astor 2018] Degrees that contain intrinsically small sets are exactly the high or DNC ones.

<u>Open Q</u> Do we get some characterization for degrees that contain computably small sets?

Notations

Def $\sigma \in 2^{<n}$, $A \in 2^{\omega}$.

$$\rho(\sigma) := \frac{|\{n : \sigma(n) = 1\}|}{n}$$

$$\underline{\rho}(A) := \liminf_{n \to \infty} \rho(A \restriction n)$$

$\overline{\rho}$ limsup

$$\rho(A) := \underline{\rho}(A) \text{ if } \underline{\rho}(A) = \overline{\rho}(A)$$

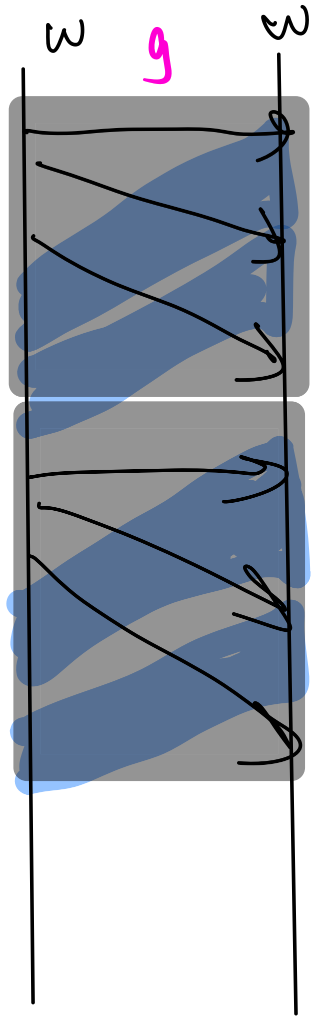
Want $A \subseteq \omega$, injective $g \leq \emptyset$ st. $\overline{\rho}(g^{-1}(A)) > 0$

and $\forall$ increasing $f \leq \emptyset$, $\overline{\rho}(f^{-1}(A)) = 0$

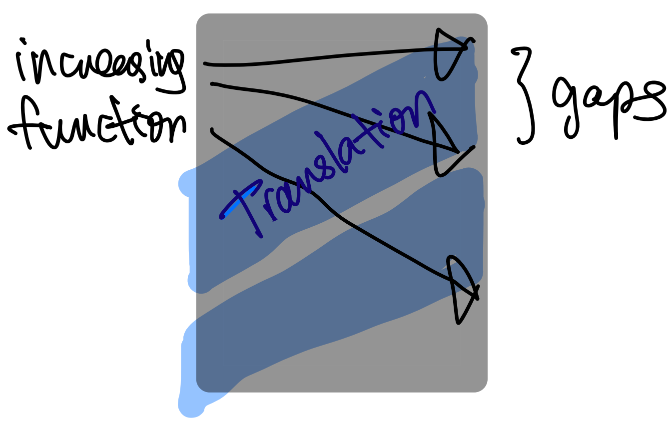
Want $A \subseteq \omega$, injective $g \leq \phi$ s.t. $\bar{\rho}(g^{-1}(A)) > 0$

and $\forall$ increasing $f \leq \phi$, $\bar{\rho}(f^{-1}(A)) = 0$

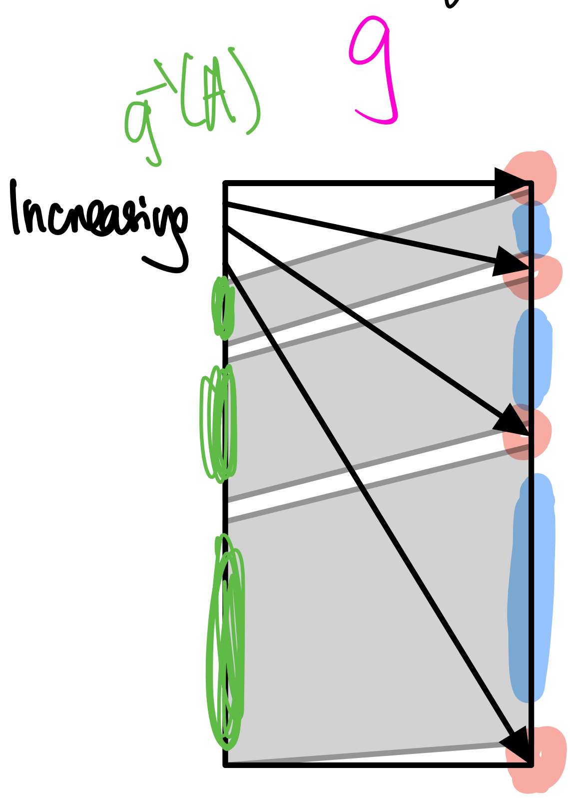
Beat single f



- g "concatenation" of disordered blocks $\boxed{\text{DB}}$
- Each DB is permutation with
  - starts as increasing function
  - blocks to fill gaps

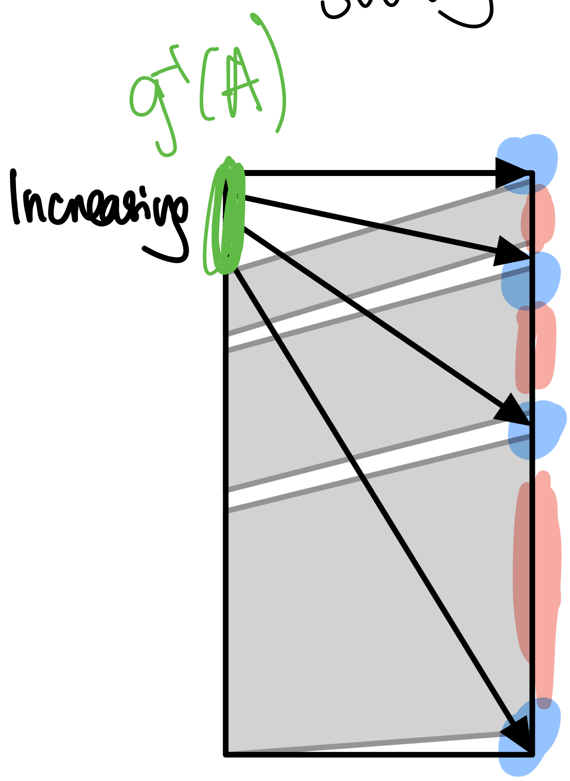


Case 1 f increases quickly



Let A be gaps between increasing part

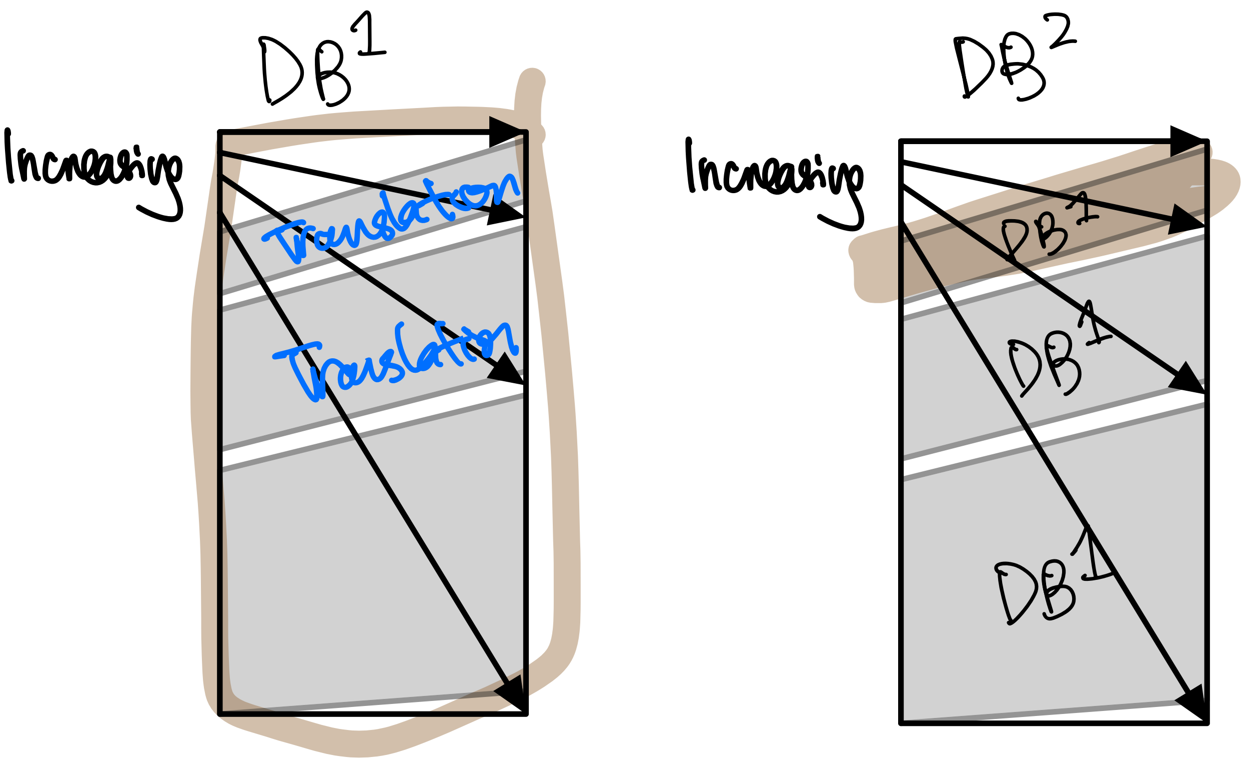
Case 2 f increases slowly



Let A be the image of inc. part

Beat f_0, f_1

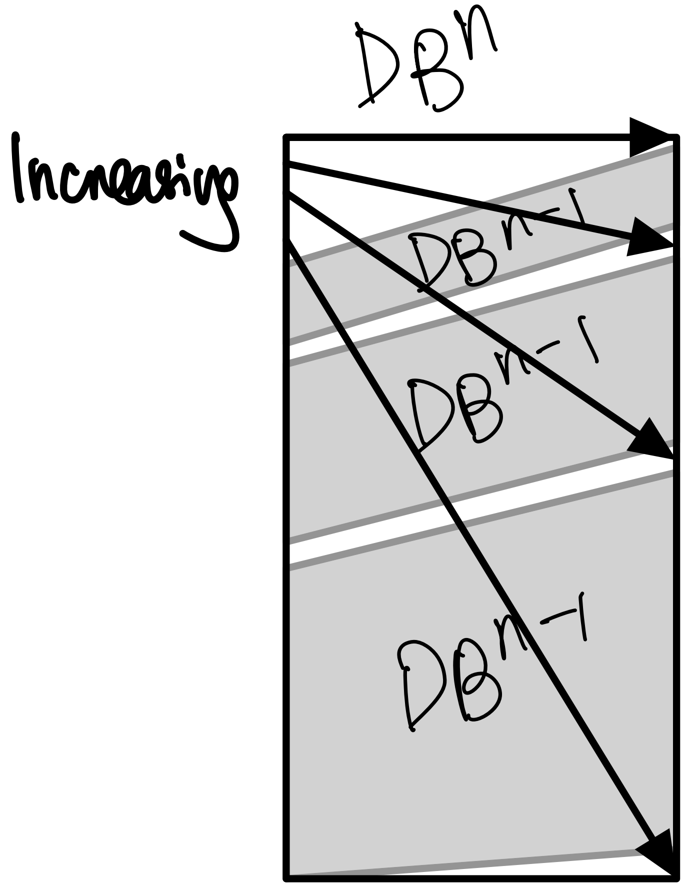
Nest disordered block DB structure



To beat single f To beat f_0, f_1

To beat n-many f's,

use n-nested (DB^n)



To beat all f's, let g be series of increasingly nested DB's.