Course: Math 535 - Mathematical Methods in Data Science (MMiDS) - Spring 2024

HWK 3

1. Find the orthogonal projection of the vector $\mathbf{v} = (4,3,0)$ onto the line spanned by $\mathbf{u} = (1,1,1)$ in $\mathbb{R}^3$.

2. Let P be a projection matrix. Show that:

a) $P^2 = P$

b) $P^T = P$

c) Check the above two claims for the projection onto the span of $\mathbf{u} = (1,0,1)$ in $\mathbb{R}^3$.

3. Let $\mathcal{Z}$ be a linear subspace of $\mathbb{R}^n$ and let $\mathbf{v}, \mathbf{w} \in \mathbb{R}^n$.

a) Show that $\|\mathrm{proj}_{\mathcal{Z}}\mathbf{v}\|_2 \leq \|\mathbf{v}\|_2$.

b) Show that $\|\mathrm{proj}_{\mathcal{Z}}\mathbf{v} - \mathrm{proj}_{\mathcal{Z}}\mathbf{w}\|_2 \leq \|\mathbf{v} - \mathbf{w}\|_2$. [Hint: Write down the geometric characterization of $\mathrm{proj}_{\mathcal{Z}}\mathbf{v}$ in terms of the vector $\mathrm{proj}_{\mathcal{Z}}\mathbf{w}$, and vice versa.]

4. Show that a matrix $A \in \mathbb{R}^{n \times m}$ with linearly independent columns can be factored into $A = QL$, where L is lower triangular. [*Hint:* Modify our procedure to obtain the QR decomposition.]

5. Suppose we consider $\mathbf{a} \in \mathbb{R}^n$ as an $n \times 1$ matrix. Write out its QR decomposition explicitly.

6. Let $R \in \mathbb{R}^{n \times m}$, with $n \geq m$, be upper triangular with non-zero entries on the diagonal. Show that the columns of R are linearly independent.